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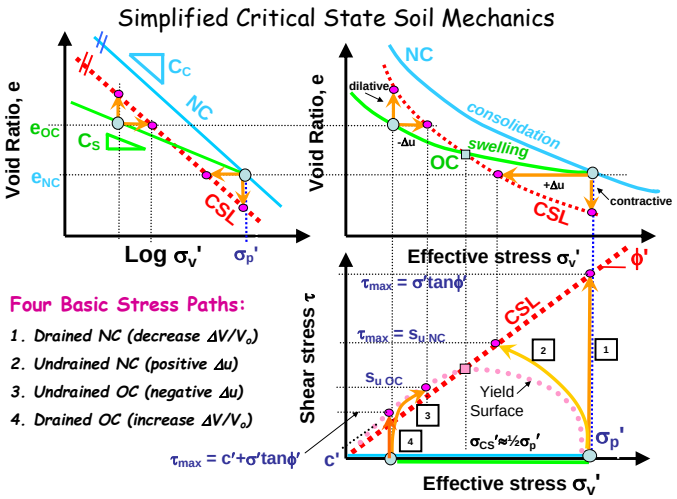
Critical-State Soil Mechanics For Dummies



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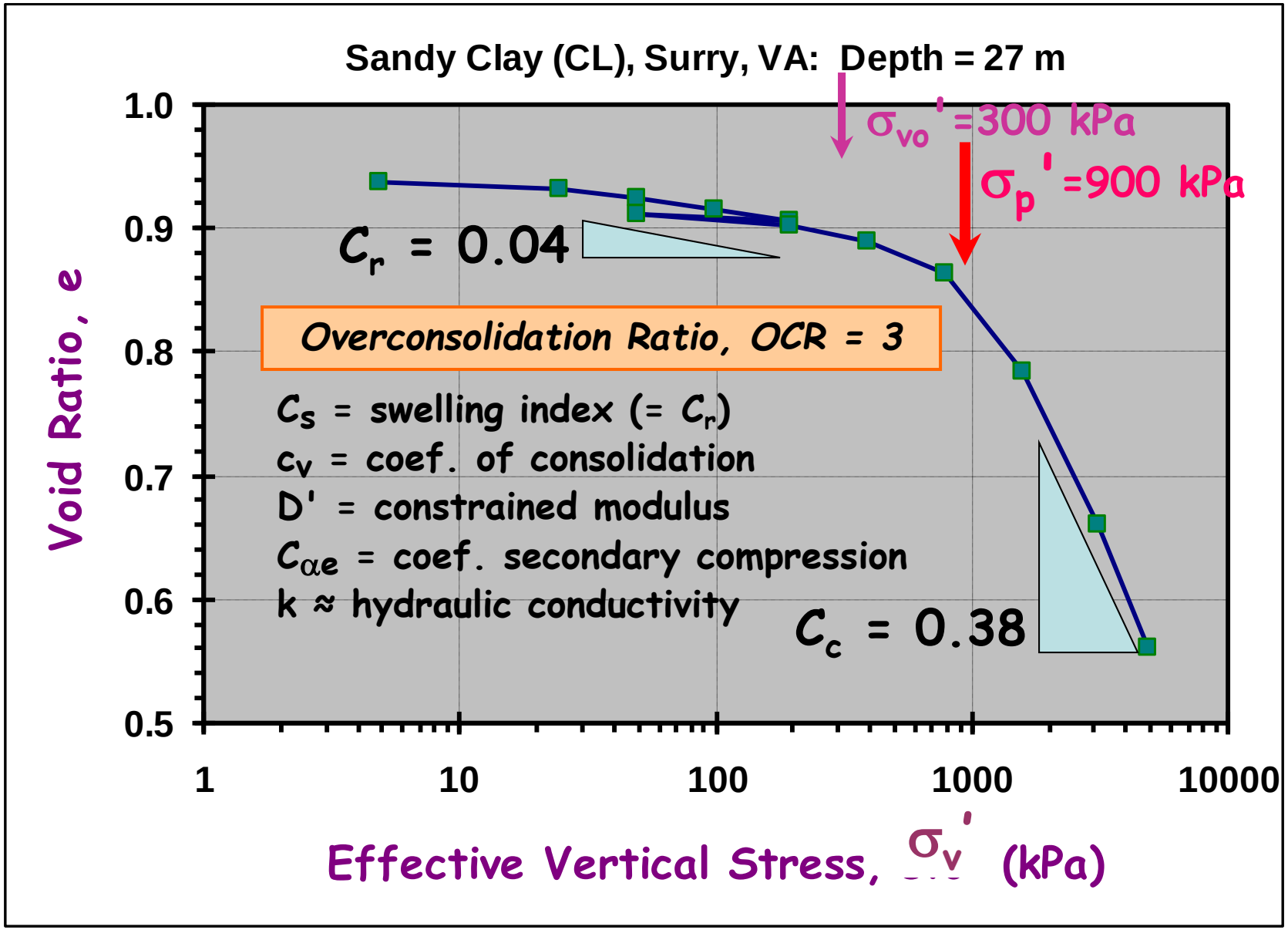
PROLOGUE

- Critical-state soil mechanics is an effective stress framework describing mechanical soil response
- In its simplest form here, we consider only shear-induced loading.
- We merely tie together two well-known concepts: (1) one-dimensional consolidation behavior, represented by e-logσ_v' curves; and (2) shear stress-vs. normal stress (τ–σ_v') from direct shear box or simple shearing (alias Mohr's circles).
- Herein, only the bare essence of CSSM concepts are presented, sufficient to describe strength & compressibility response.

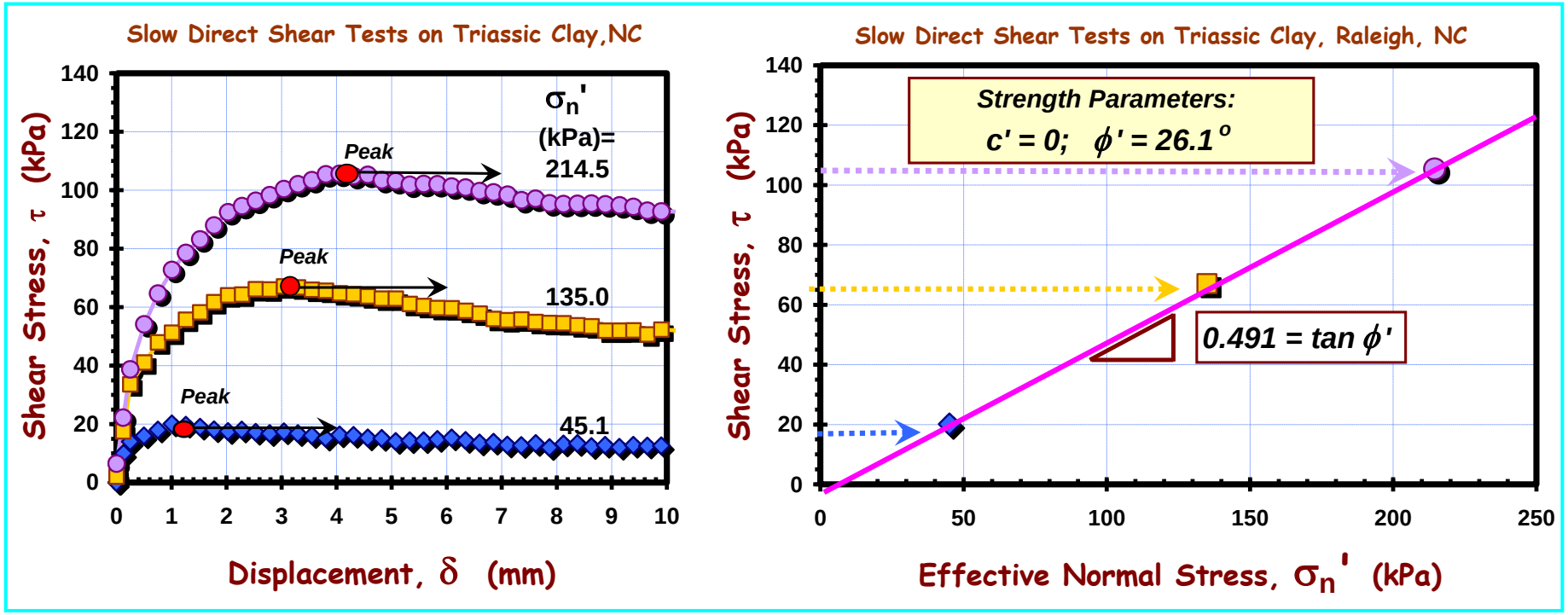
Critical State Soil Mechanics (CSSM)

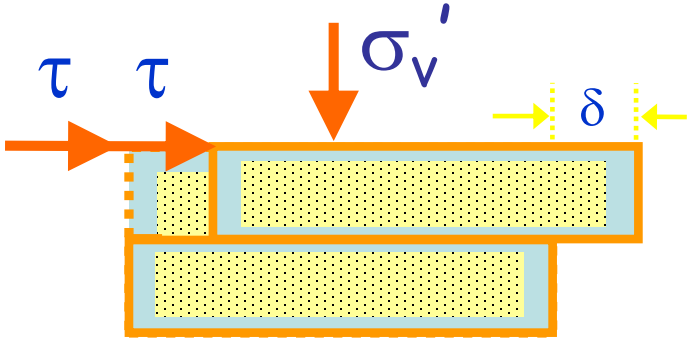
- Experimental evidence provided by Hvorslev (1936; 1960, ASCE); Henkel (1960, ASCE Boulder) Henkel & Sowa (1961, ASTM STP 361)
- Mathematics presented elsewhere, including: Schofield & Wroth (1968); Burland (1968); Wood (1990).
- In basic form: 3 material constants (ϕ', C_c, C_s) plus initial state (e_0, σ_{vo}', OCR)
- Constitutive Models, include: Original Cam-Clay, Modified Cam Clay, NorSand, Bounding Surface, MIT-E3 (Whittle, 1993) & MIT-S1 (Pestana) and others (Adachi, Oka, Ohta, Dafalias)
- "Undrained" is just one specific stress path
- Yet !!! CSSM is missing from most textbooks and undergrad & grad curricula in the USA.

One-Dimensional Consolidation

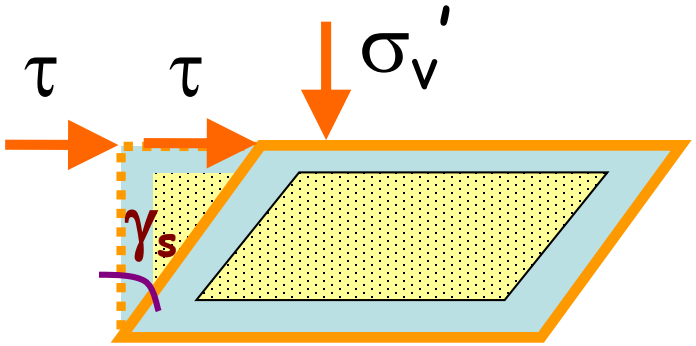


Direct Shear Test Results



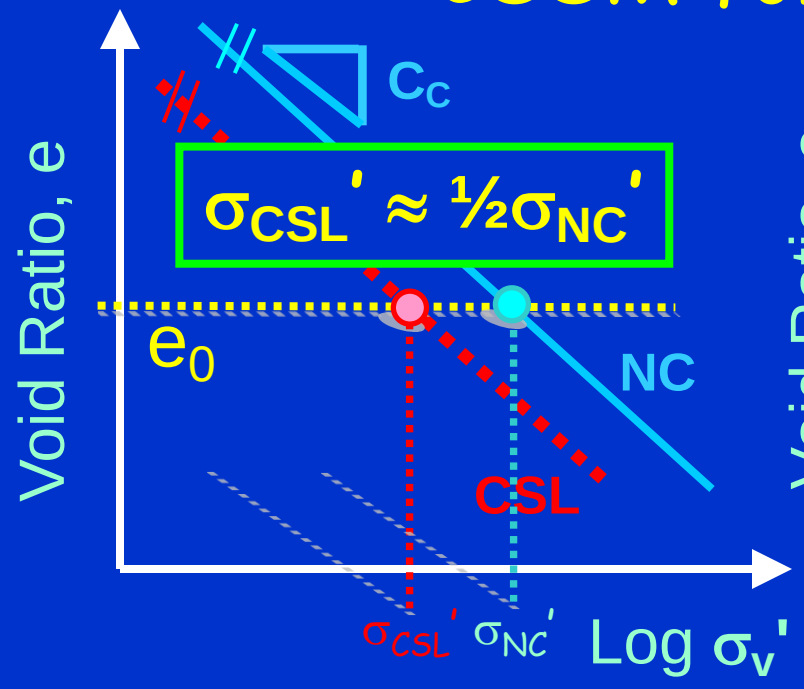


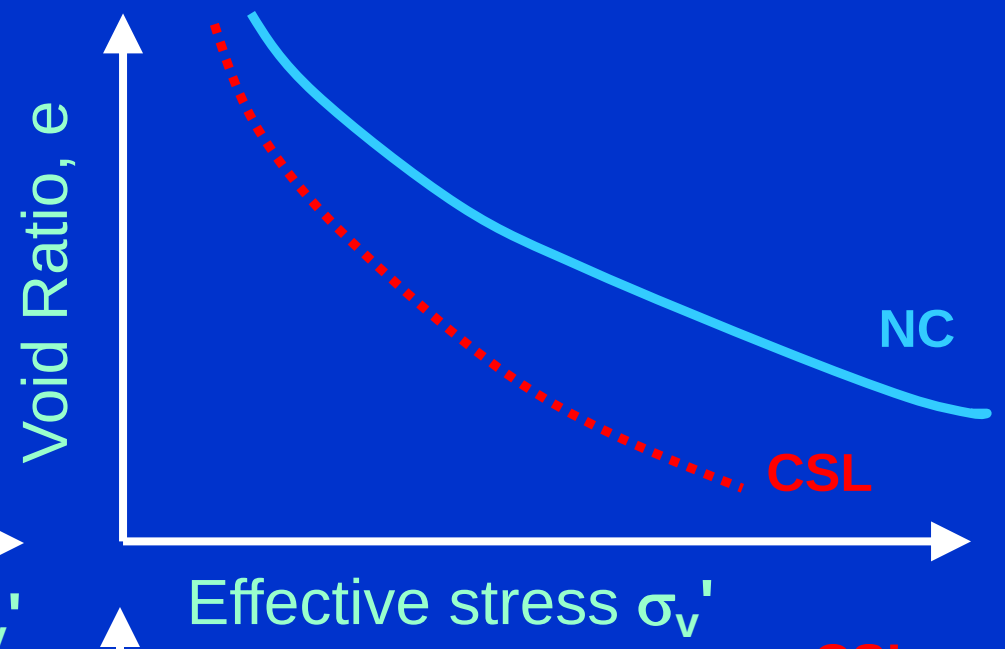
Direct Shear Box (DSB)



Direct Simple Shear (DSS)

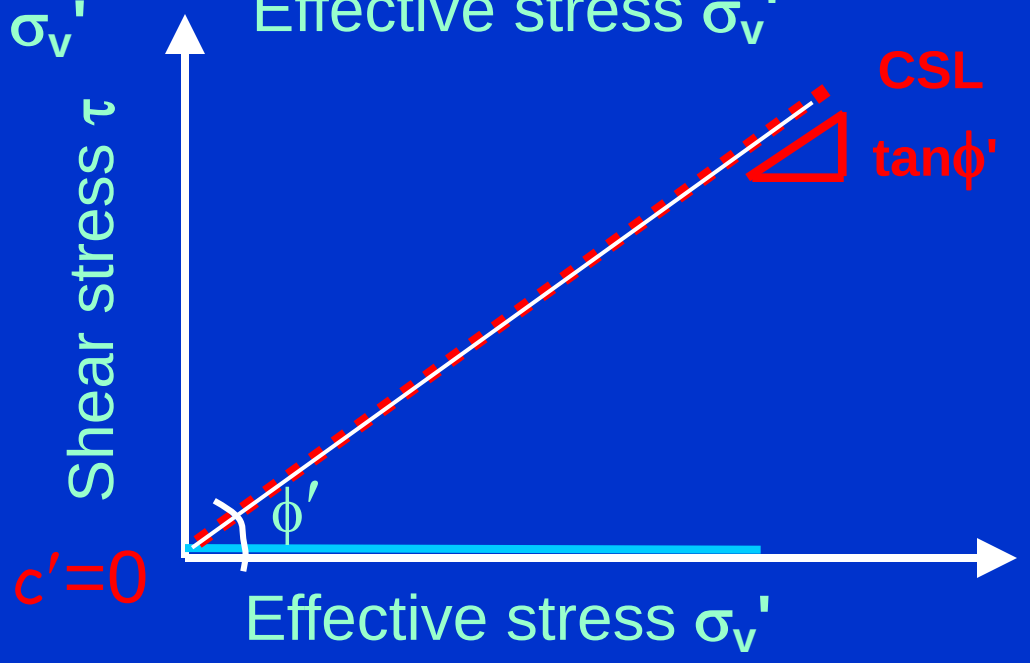
CSSM for Dummies



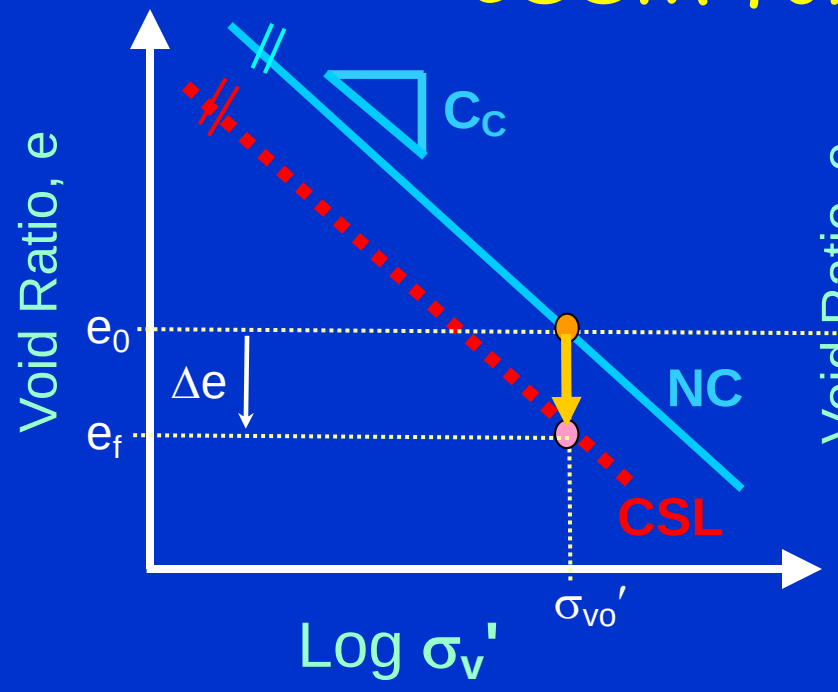


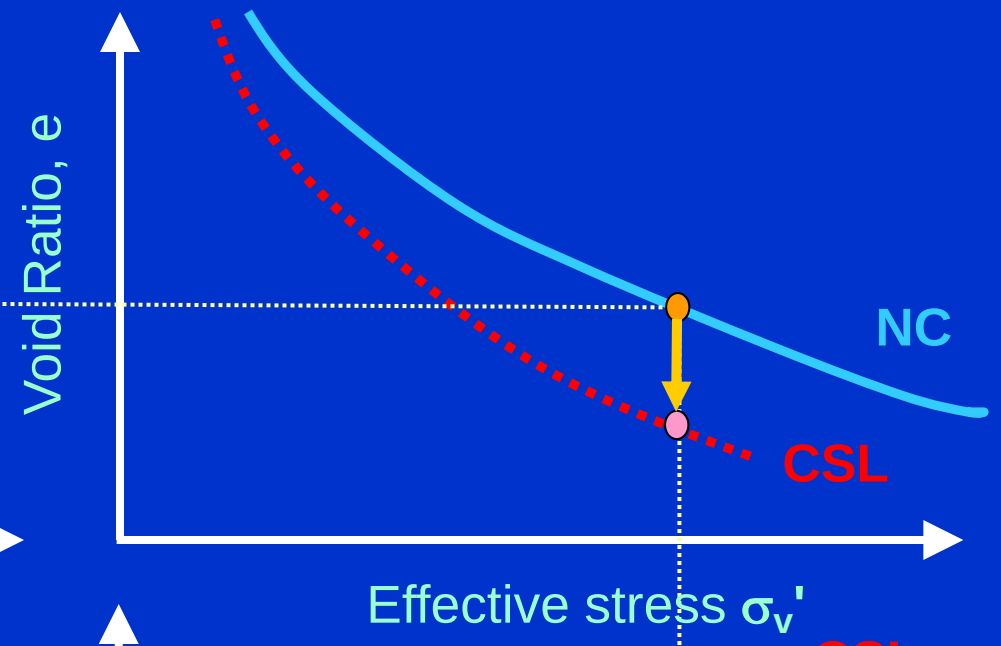
CSSM Premise:

"All stress paths fail on the critical state line (CSL)"



CSSM for Dummies





STRESS PATH No.1

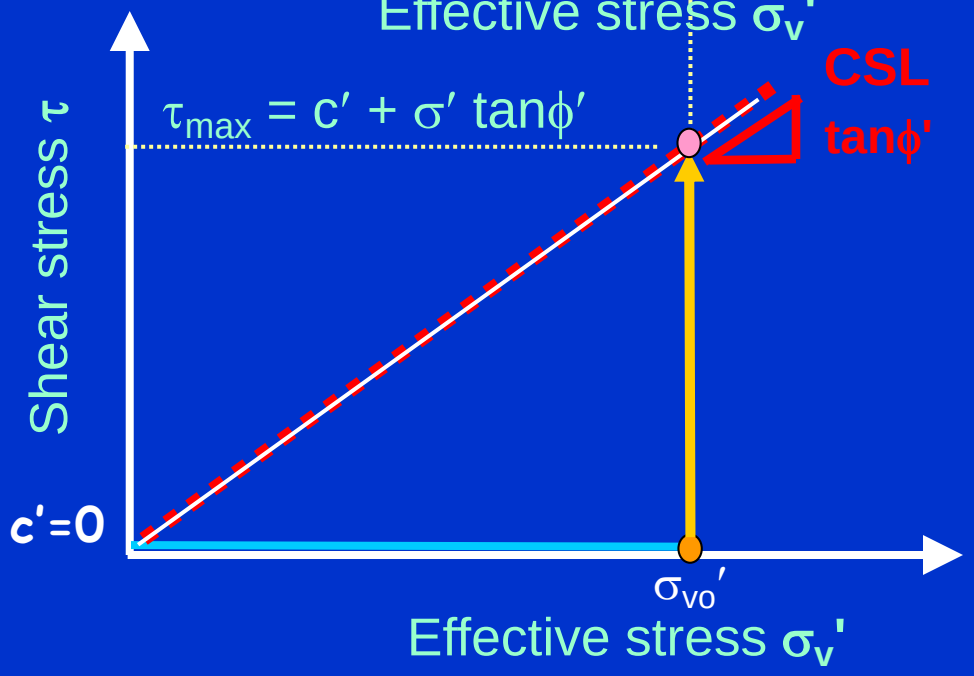
NC Drained Soil

Given: e_0, σ_{vo}', NC (OCR=1)

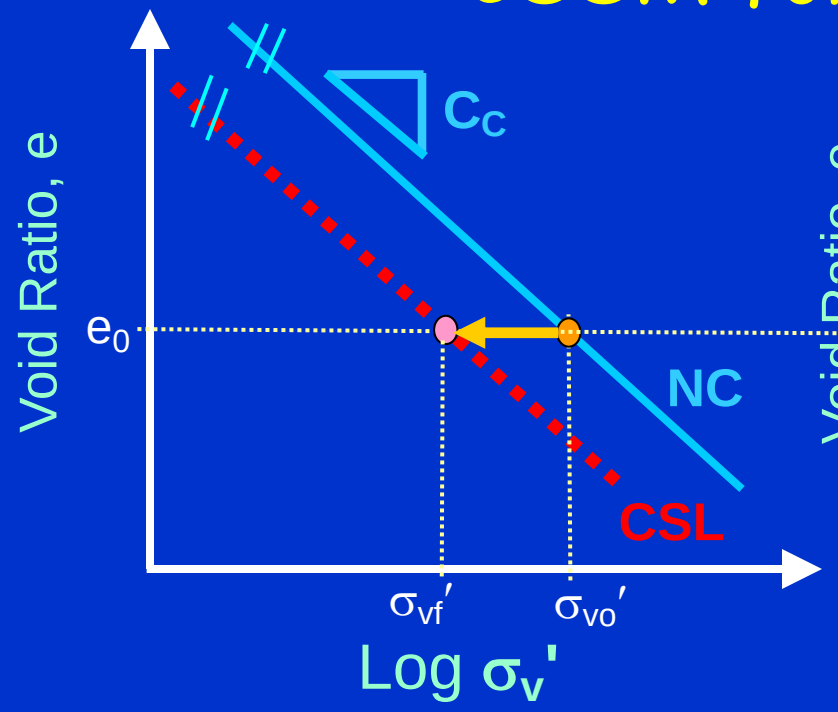
Drained Path: $\Delta u = 0$

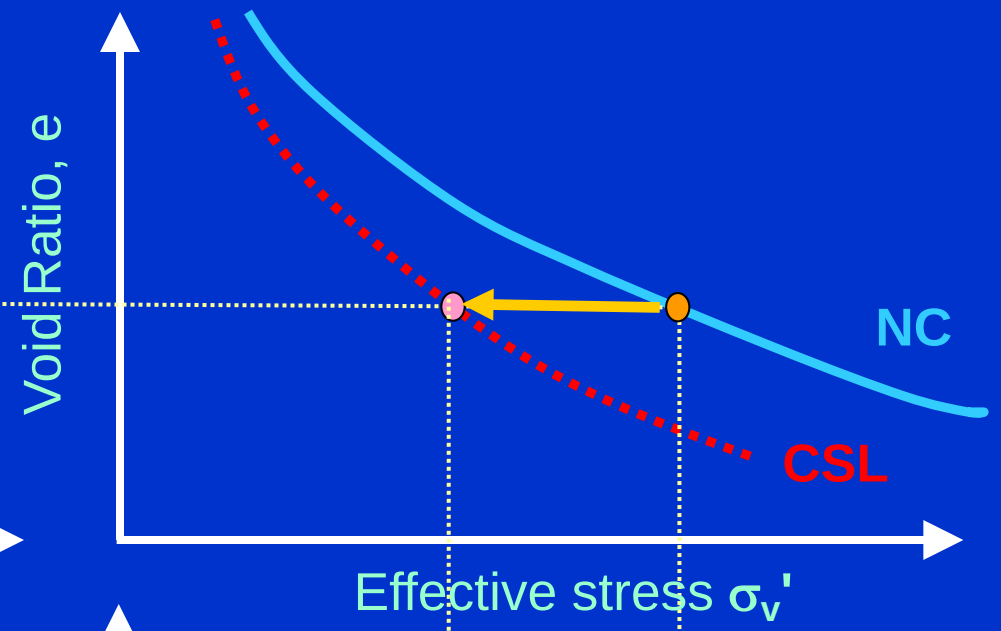
Volume Change is Contractive:

$\varepsilon_{vol} = \Delta e/(1+e_0) < 0$



CSSM for Dummies





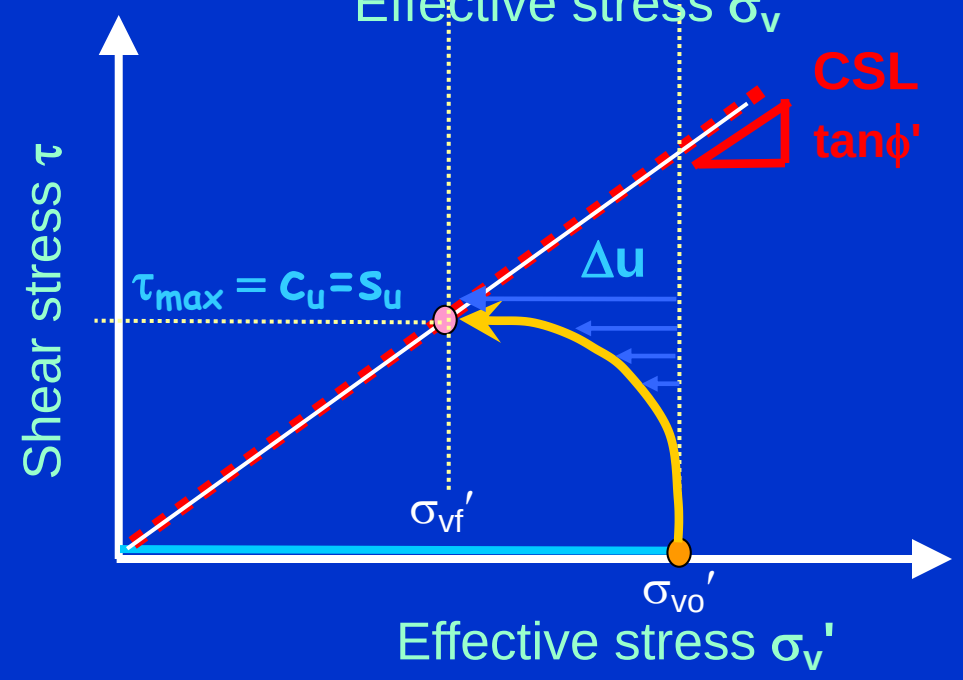
STRESS PATH No.2

NC Undrained Soil

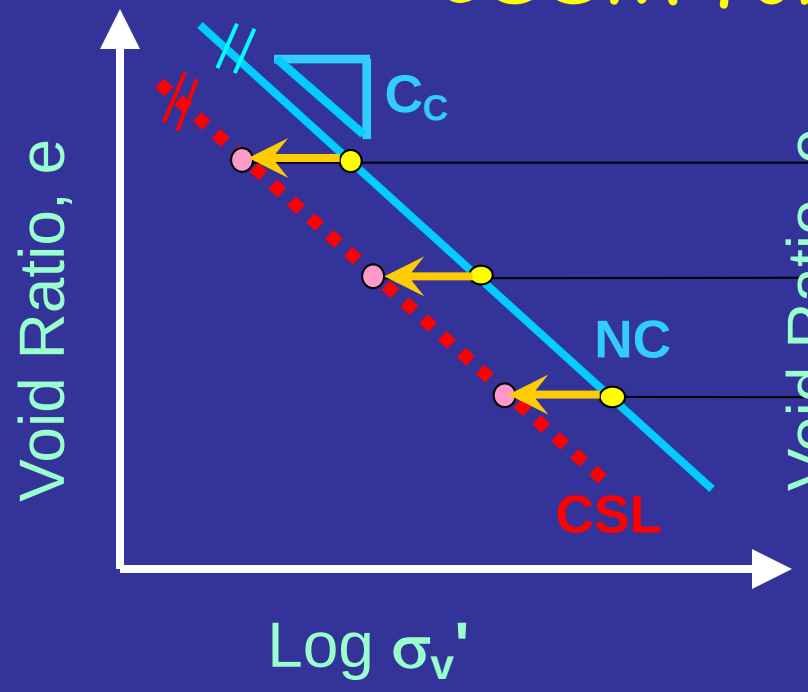
Given: e_0, σ_{vo}', NC (OCR=1)

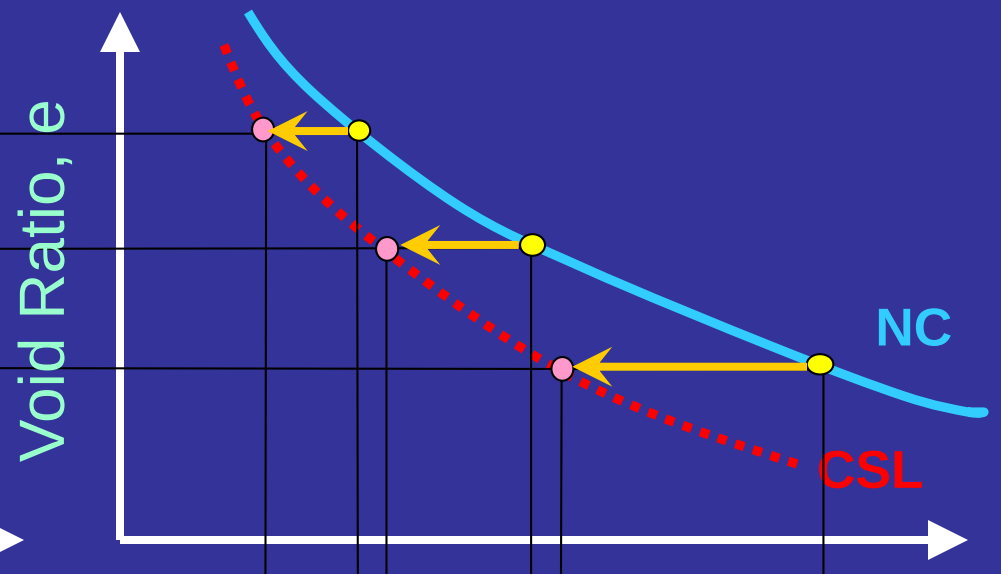
Undrained Path: $\Delta V/V_0 = 0$

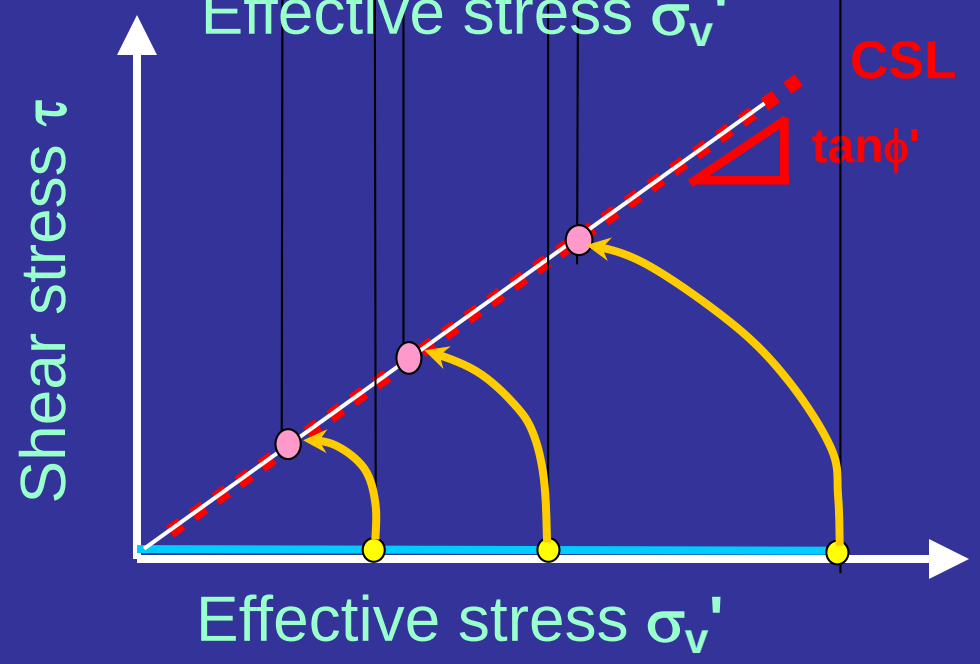
$+\Delta u$ = Positive Excess Porewater Pressures



CSSM for Dummies





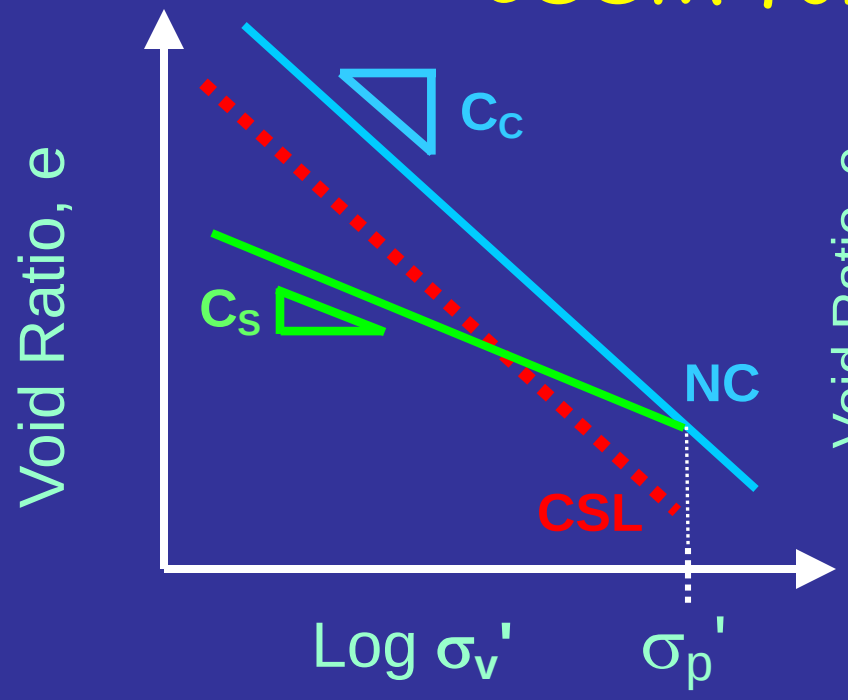


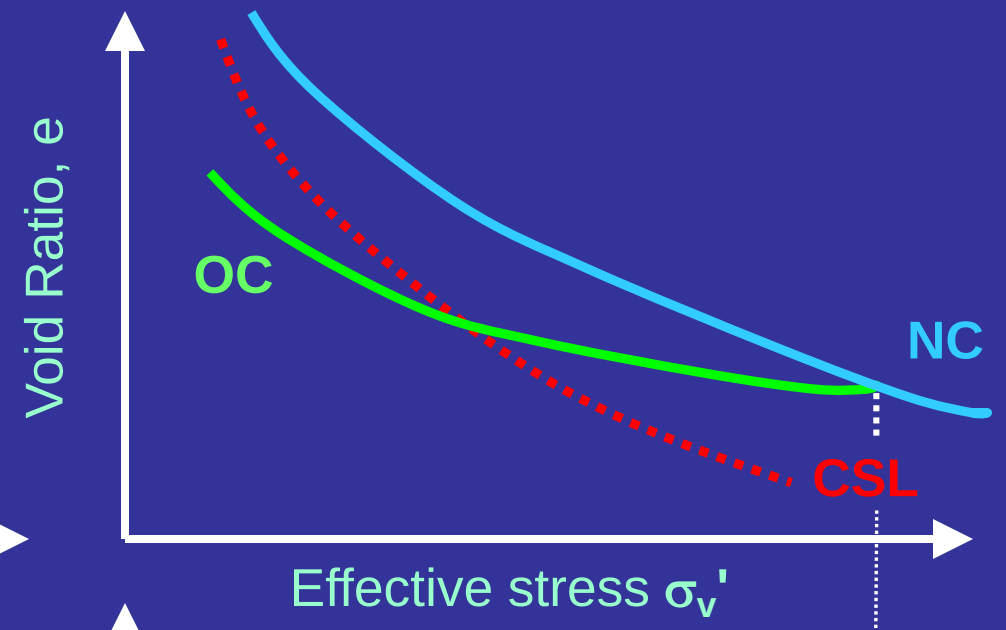
Note: All NC undrained stress paths are parallel to each other, thus:

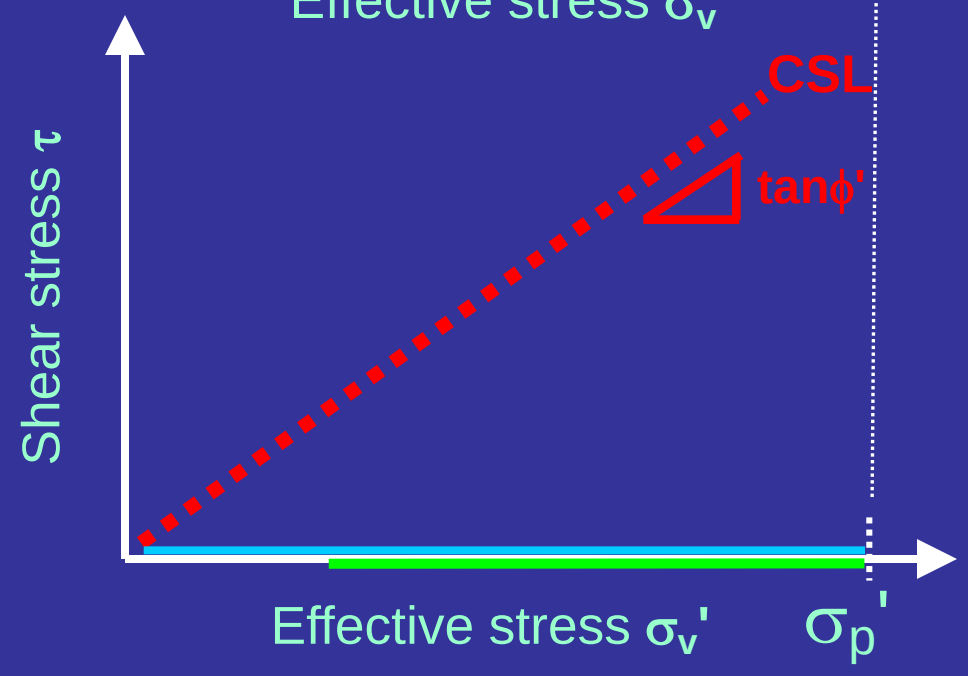
s_u/σ_{vo}' = constant

DSS: $s_u/\sigma_{vo}'_{NC} = \frac{1}{2}\sin\phi'$

CSSM for Dummies







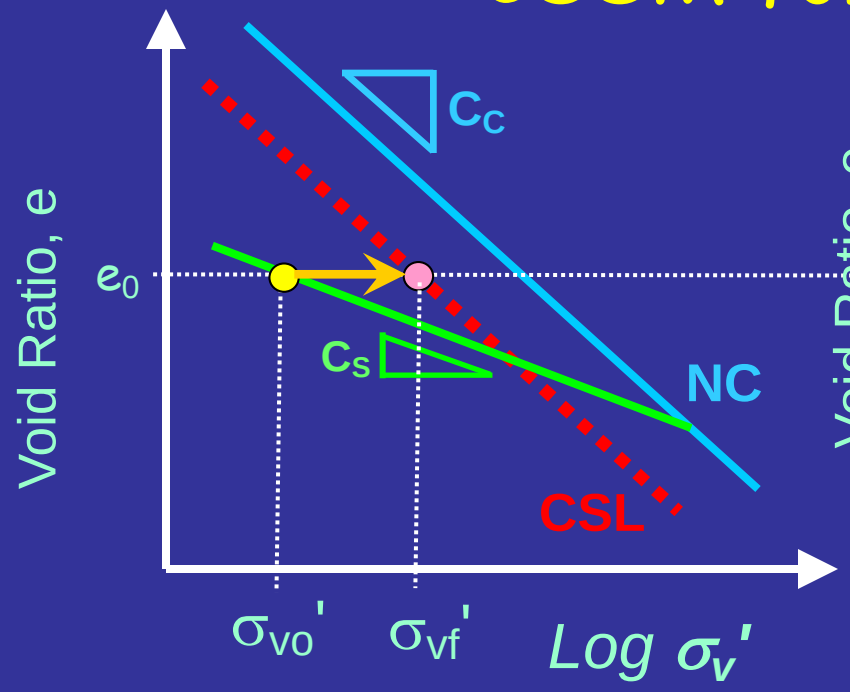
Overconsolidated States:

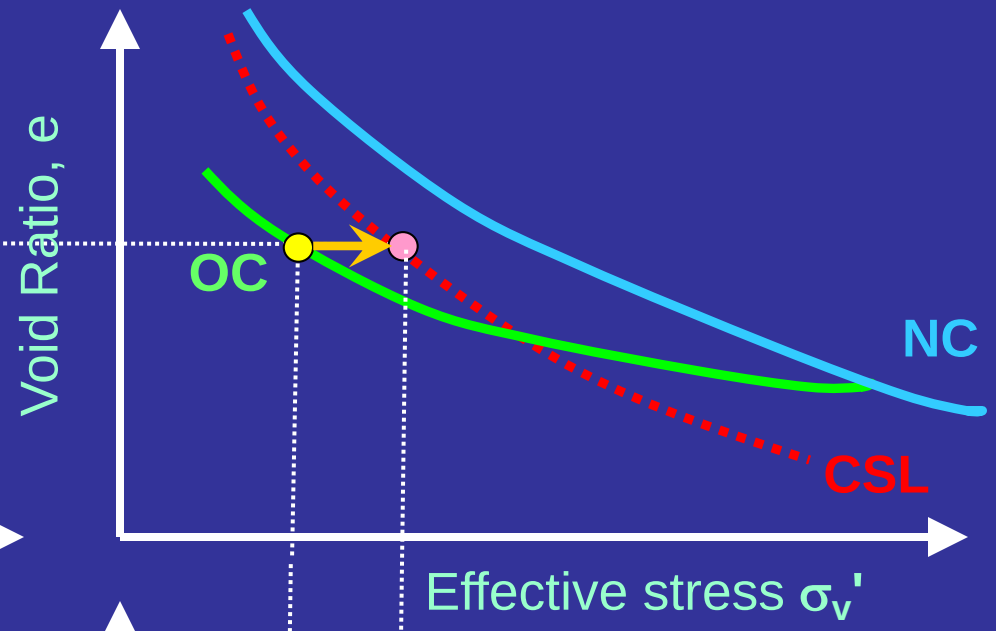
e_0, σ_{vo}', and OCR = σ_p'/σ_{vo}'

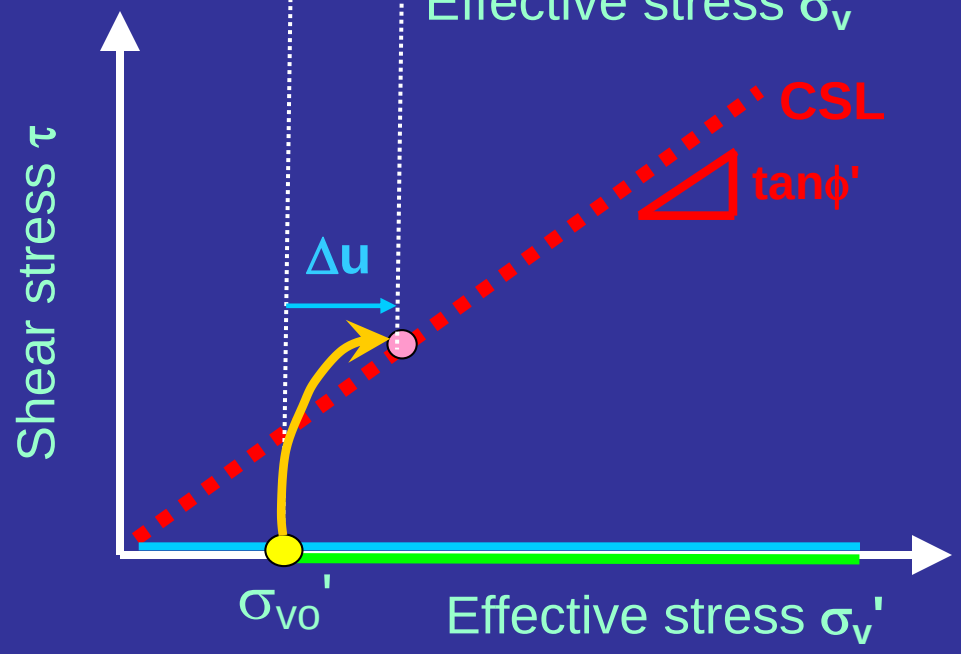
where σ_p' = σ_{vmax}' = P_c' = preconsolidation stress;

OCR = overconsolidation ratio

CSSM for Dummies







Stress Path No. 3

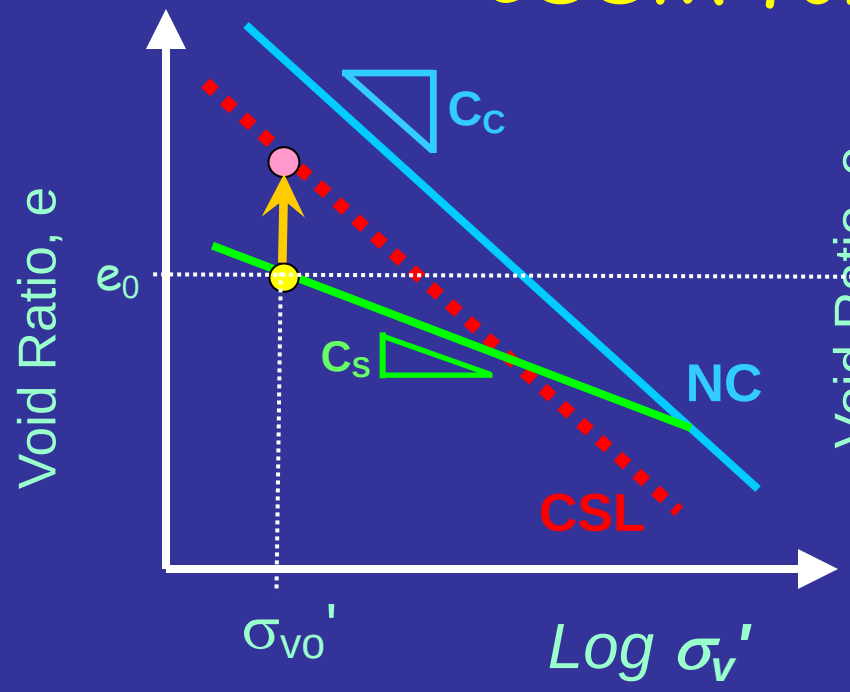
Undrained OC Soil:

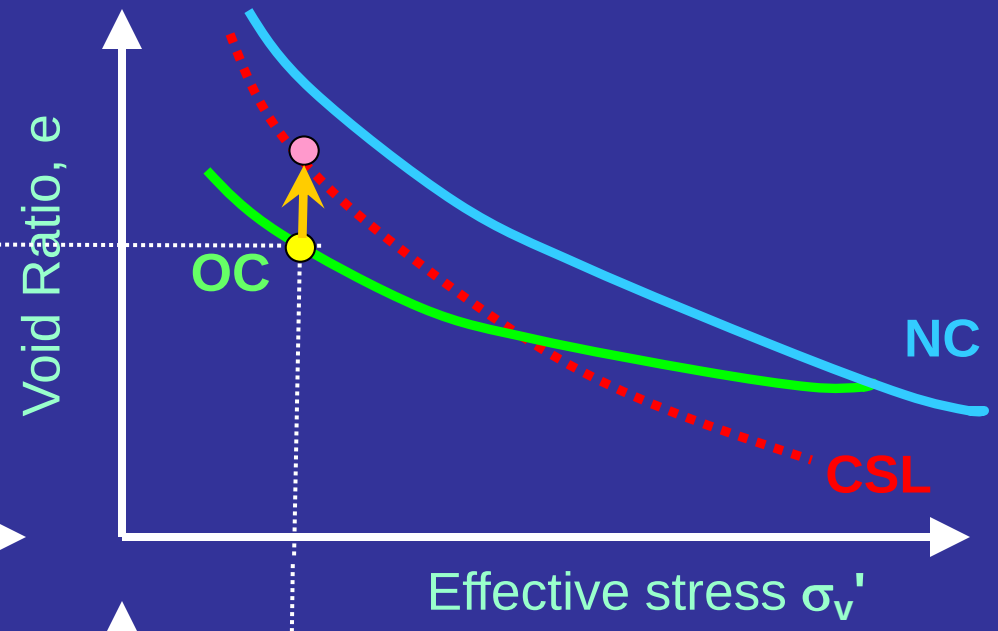
e_0, σ_{vo}', and OCR

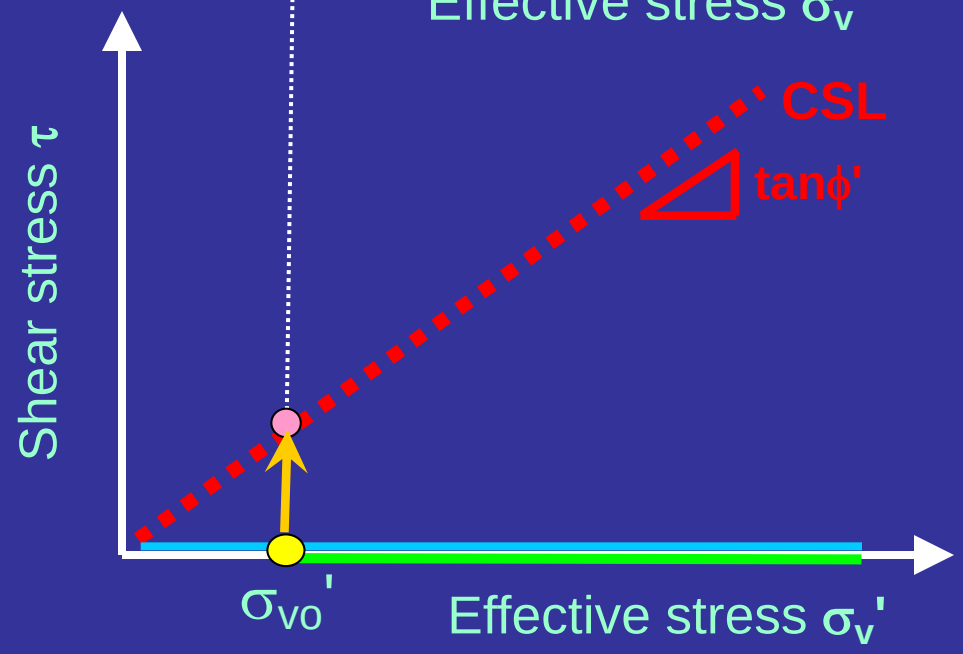
Stress Path: $\Delta V/V_0 = 0$

Negative Excess Δu

CSSM for Dummies







Stress Path No. 4

Drained OC Soil:

e_0, σ_{vo}', and OCR

Stress Path: $\Delta u = 0$

Dilatancy: $\Delta V/V_0 > 0$

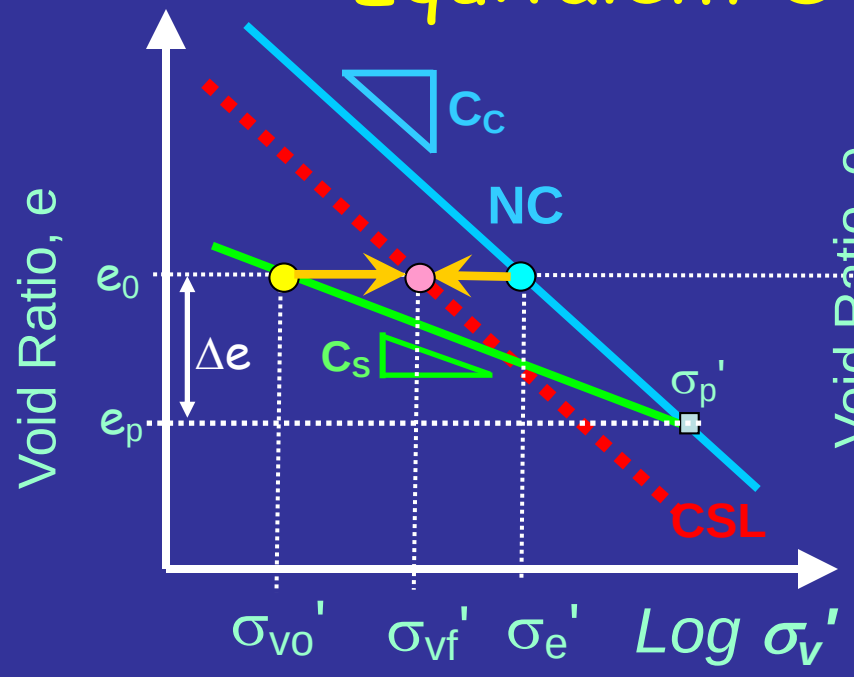
Critical state soil mechanics

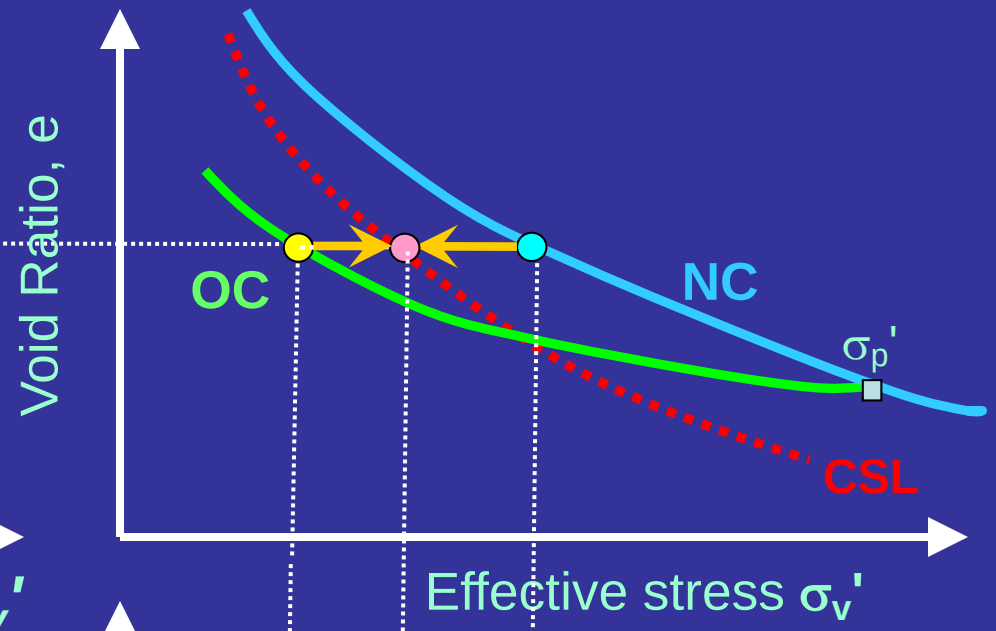
- Initial state: e_0, σ_{vo}', and $OCR = \sigma_p'/\sigma_{vo}'$
- Soil constants: ϕ', C_c, and C_s $(\Lambda = 1-C_s/C_c)$
- For NC soil (OCR =1):
  - Undrained ($\varepsilon_{vol} = 0$): $+\Delta u$ and $\tau_{max} = s_u = c_u$
  - Drained ($\Delta u = 0$) and contractive (decrease ε_{vol})

  For OC soil:
  - Undrained ($\varepsilon_{vol} = 0$): $-\Delta u$ and $\tau_{max} = s_u = c_u$
  - Drained ($\Delta u = 0$) and dilative (Increase ε_{vol})

There's more ! Semi-drained, Partly undrained, Cyclic…..

Equivalent Stress Concept

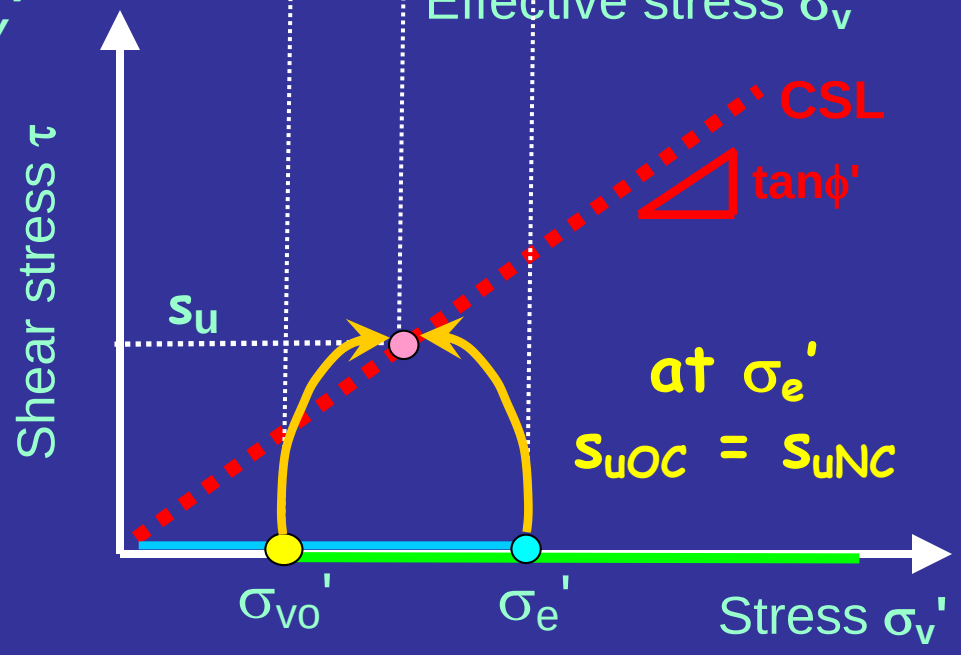




1. OC State (e_o, σ_{vo}', σ_p')
2. Project OC state to NC line for equivalent stress, σ_e'

$$\Delta e = C_s \log(\sigma_p'/\sigma_{vo}')$$
$$\Delta e = C_c \log(\sigma_e'/\sigma_p')$$

3. $\sigma_e' = \sigma_{vo}' \, OCR^{[1-Cs/Cc]}$



Critical state soil mechanics

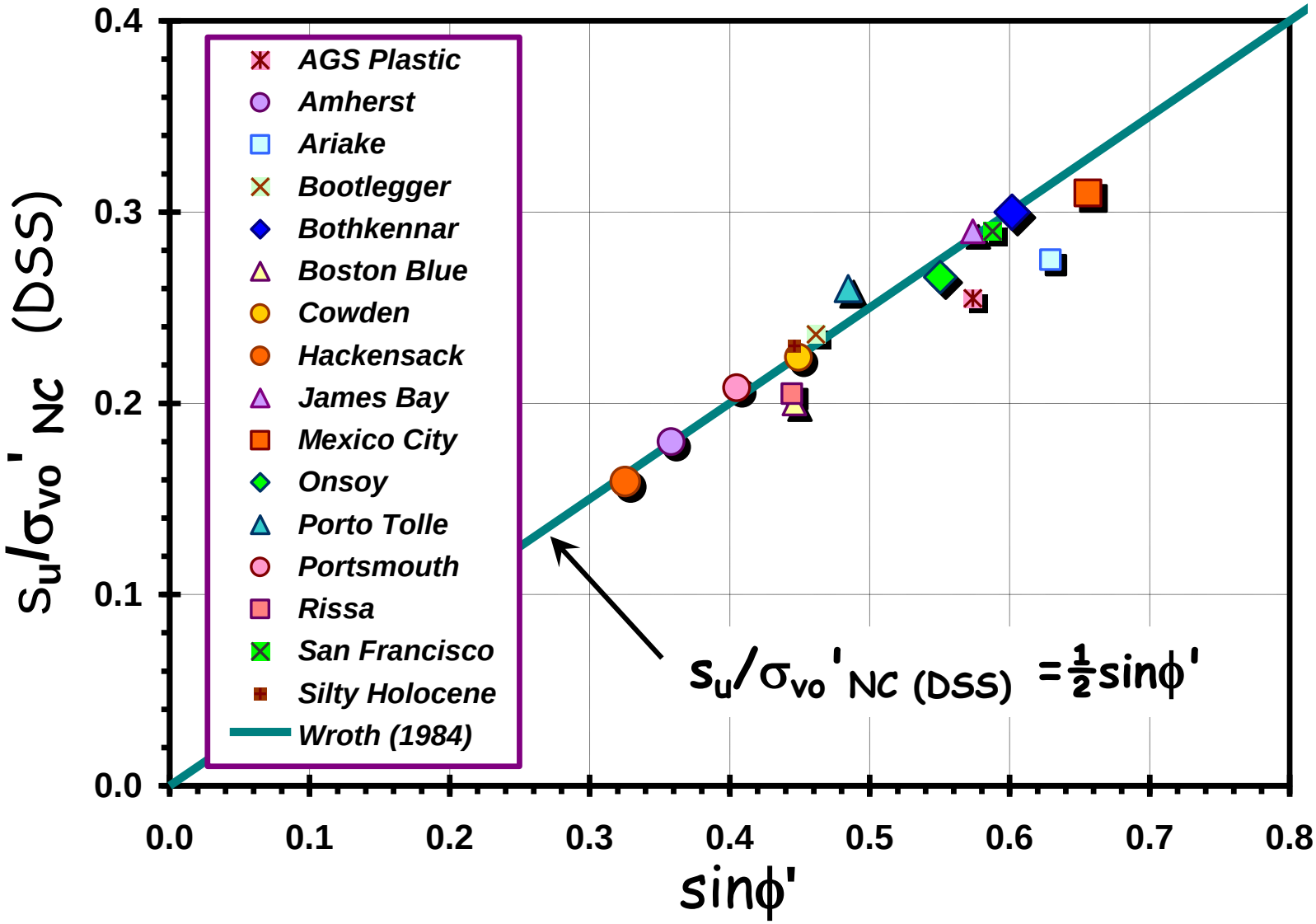
- Previously: $s_u/\sigma_{vo}' = \text{constant}$ for NC soil
- On the virgin compression line: $\sigma_{vo}' = \sigma_e'$
- Thus: $s_u/\sigma_e' = \text{constant}$ for all soil (NC & OC)
- For simple shear: $s_u/\sigma_e' = \frac{1}{2}\sin\phi'$
- Equivalent stress: $\sigma_e' = \sigma_{vo}'\ OCR^{[1-Cs/Cc]}$

Normalized Undrained Shear Strength:

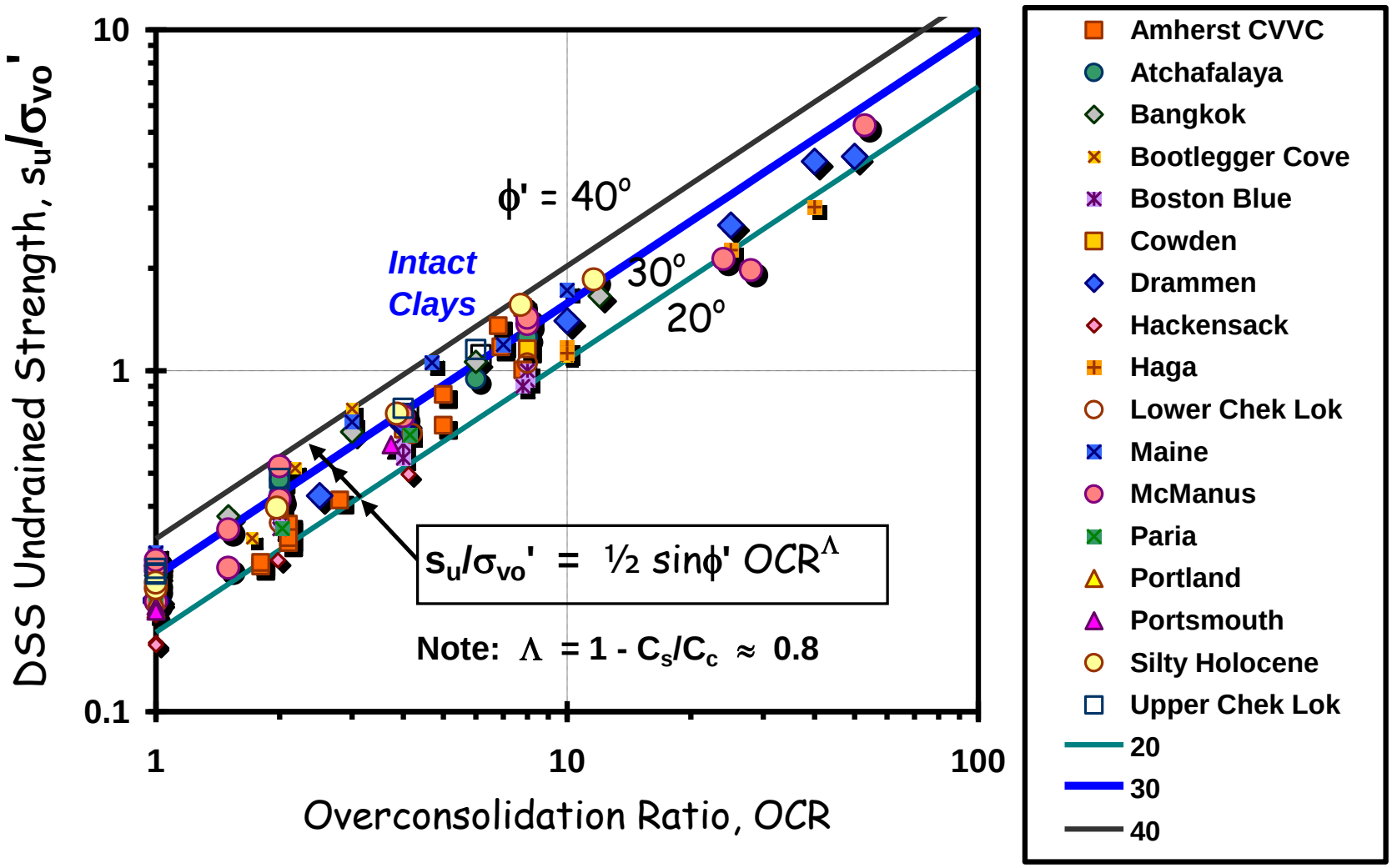
$$s_u/\sigma_{vo}' = \tfrac{1}{2}\sin\phi'\ OCR^{\Lambda}$$

where $\Lambda = (1-C_s/C_c)$

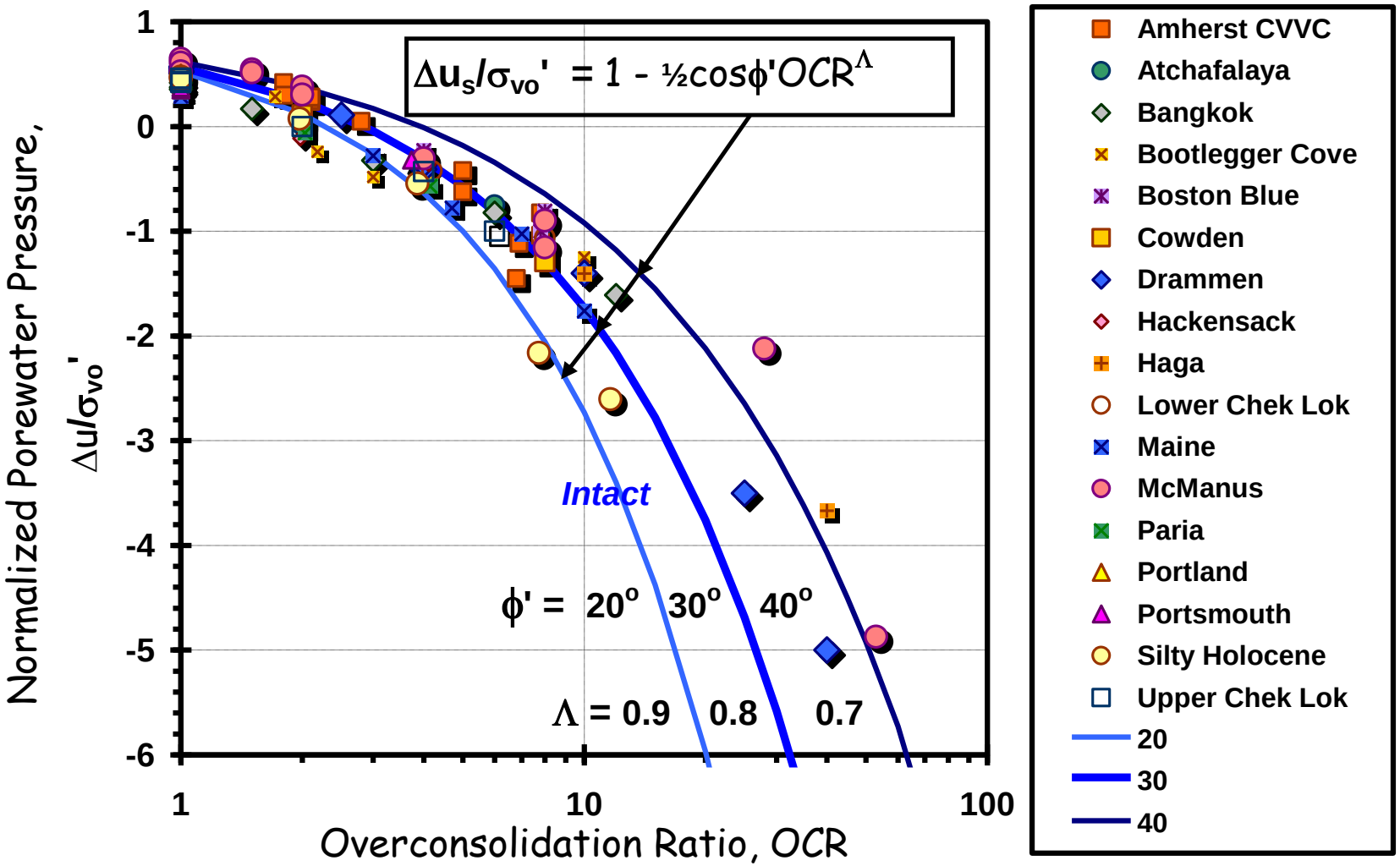
Undrained Shear Strength from CSSM



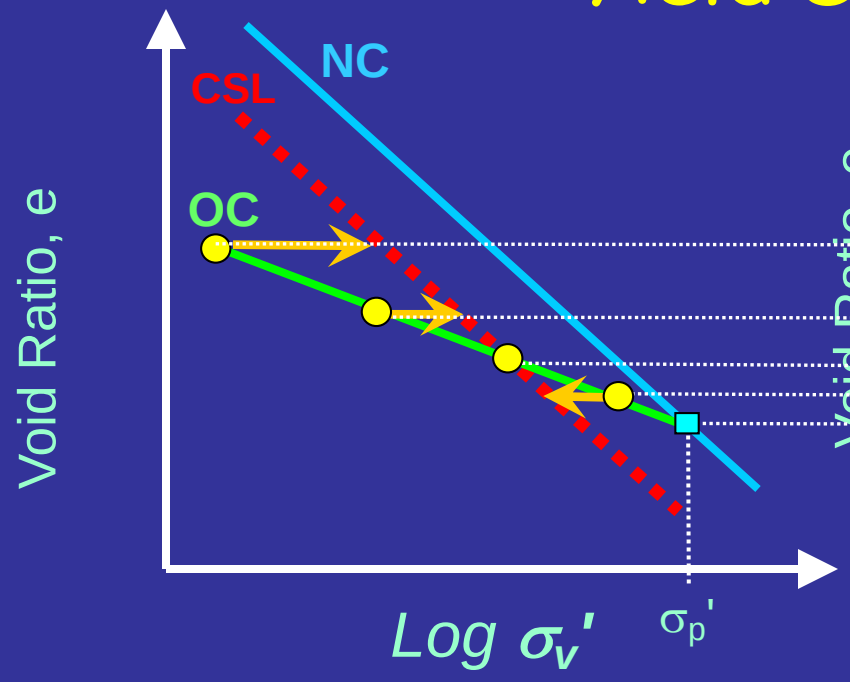
Undrained Shear Strength from CSSM

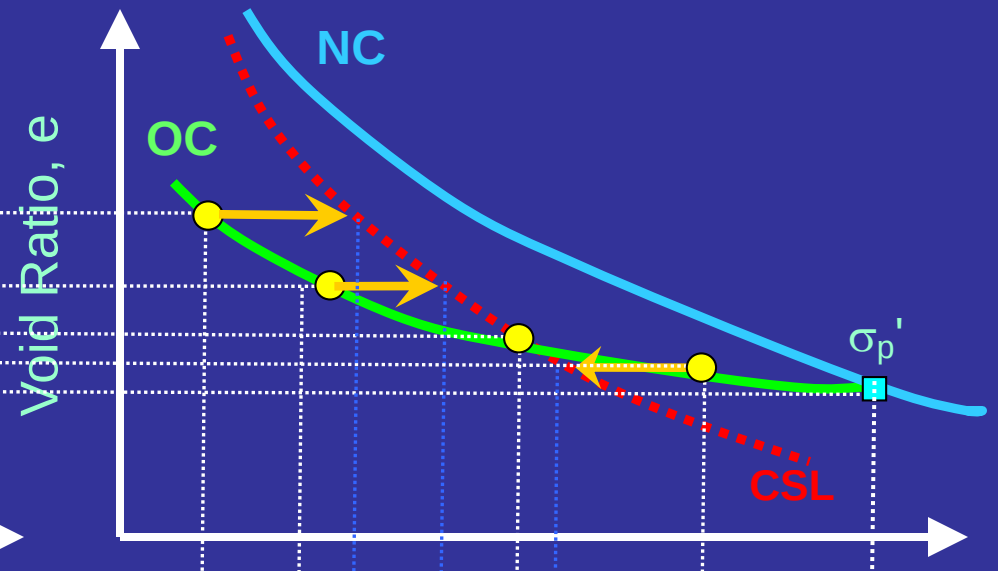


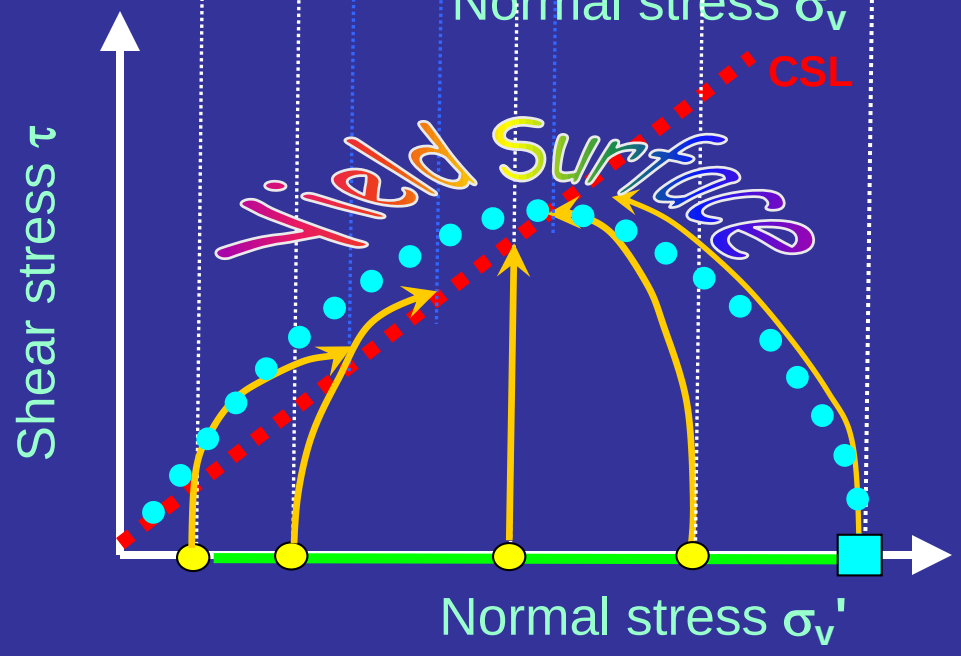
Porewater Pressure Response from CSSM



Yield Surfaces

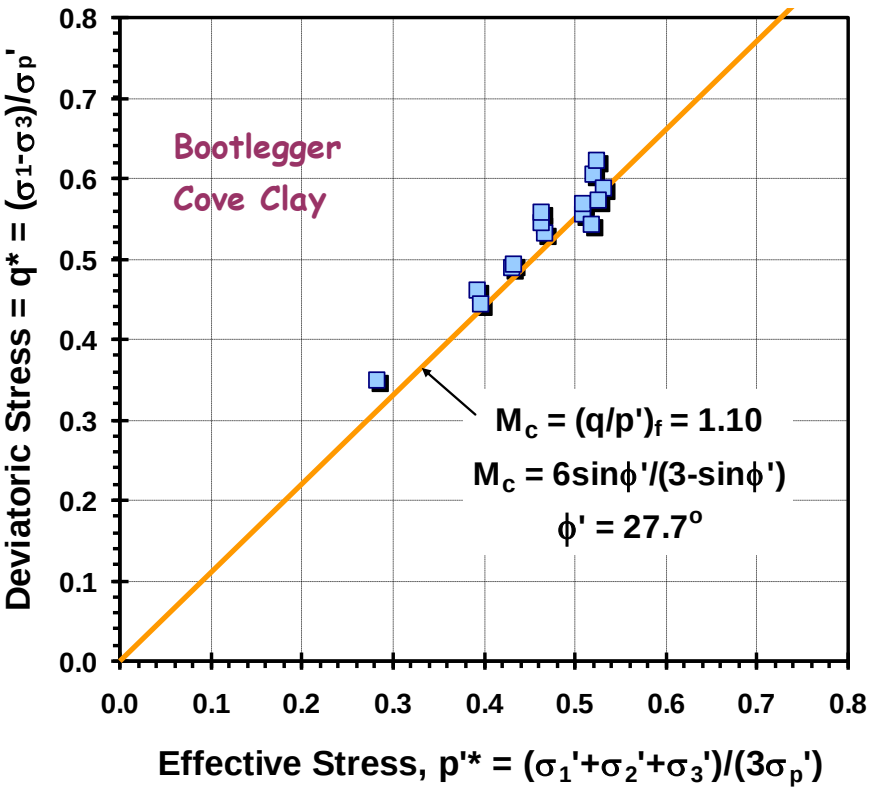


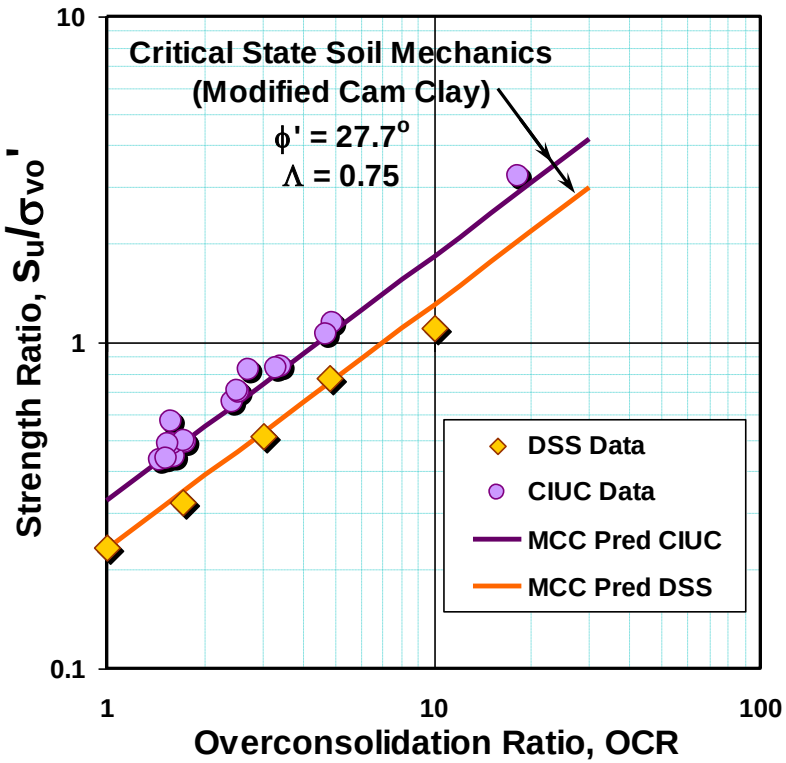




- Yield surface represents 3-d preconsolidation
- Quasi-elastic behavior within the yield surface

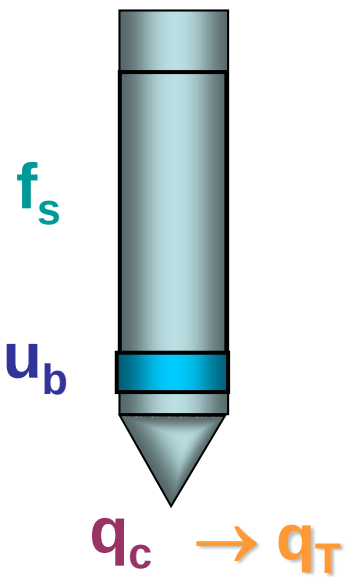
Port of Anchorage, Alaska

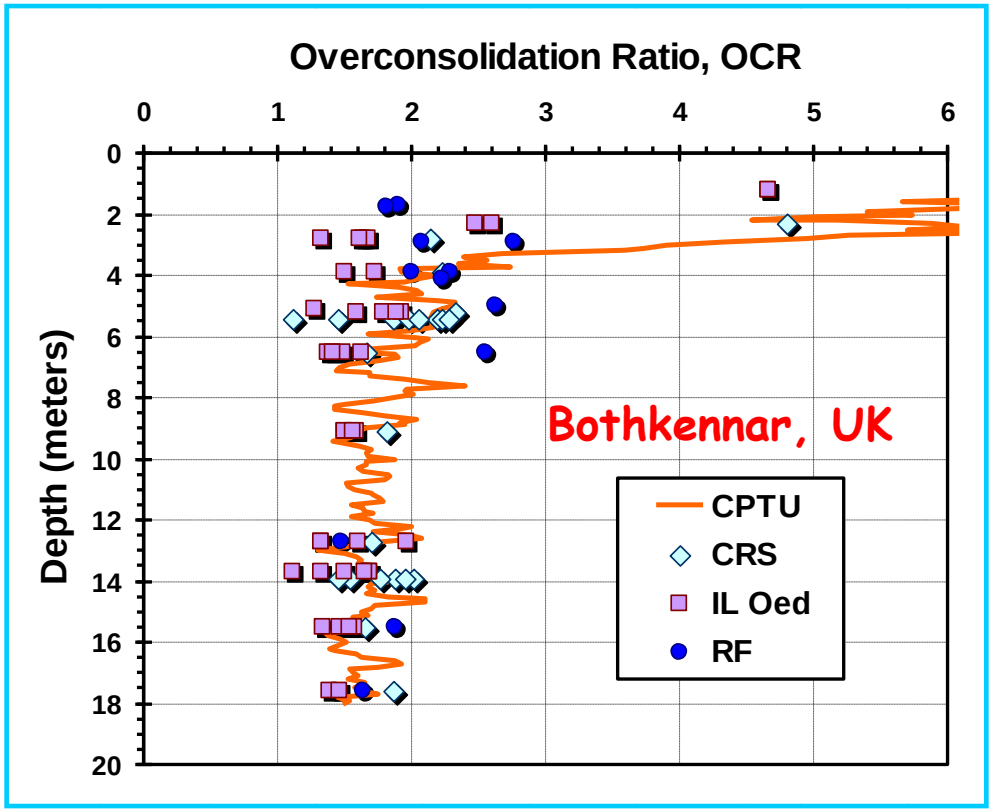




Cavity Expansion – Critical State Model for Evaluating OCR in Clays from Piezocone Tests

$$OCR = 2\left[\frac{1}{1.95M+1}\left(\frac{q_T - u_b}{\sigma_{vo}'}\right)\right]^{1/\Lambda}$$

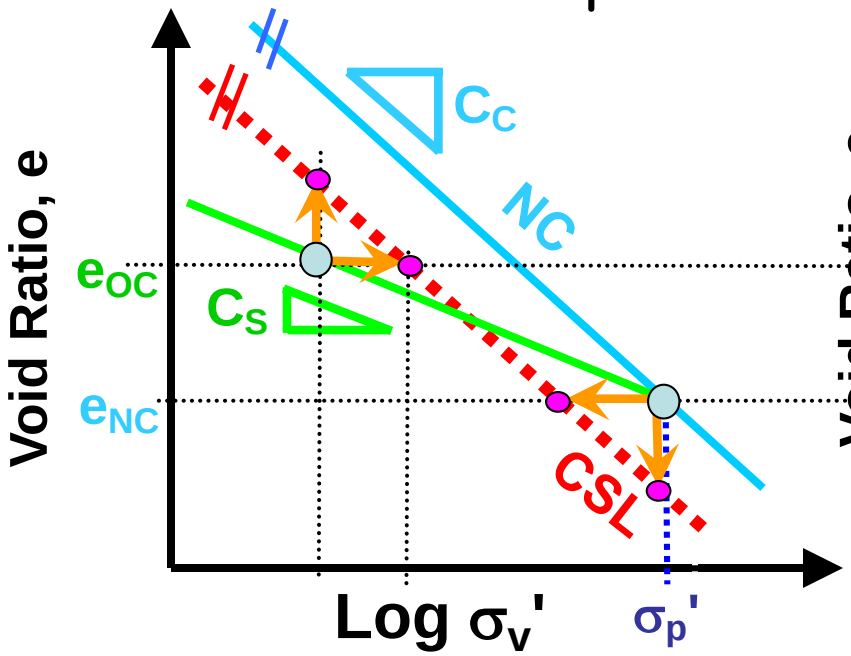


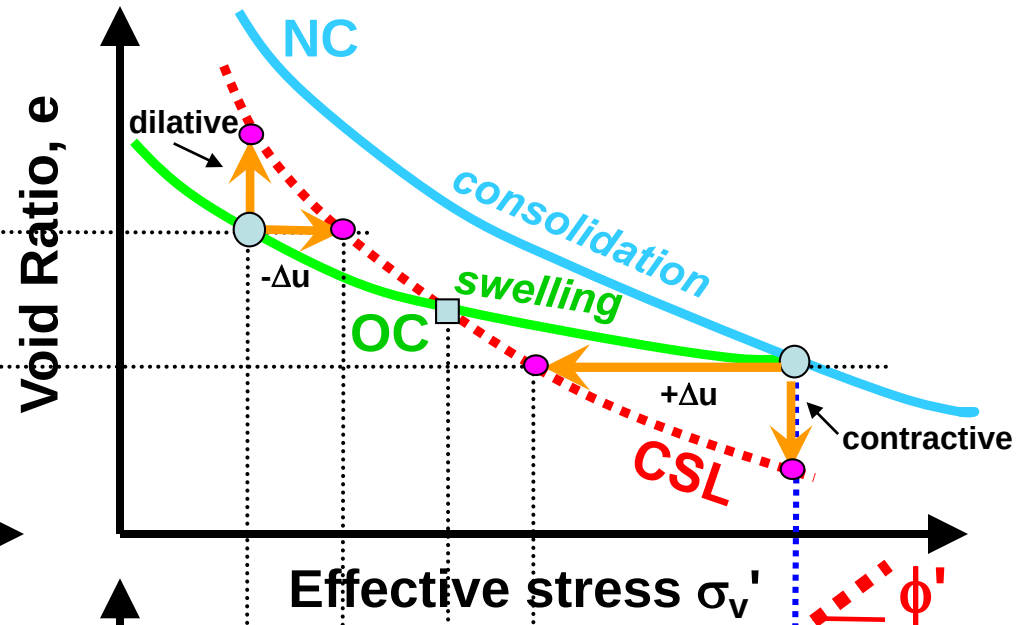


Critical state soil mechanics

- Initial state: e_0, σ_{vo}', and $OCR = \sigma_p'/\sigma_{vo}'$
- Soil constants: ϕ', C_c, and C_s $(\Lambda = 1-C_s/C_c)$
- Using effective stresses, CSSM addresses:
  - NC and OC behavior
  - Undrained vs. Drained (and other paths)
  - Positive vs. negative porewater pressures
  - Volume changes (contractive vs. dilative)
  - $s_u/\sigma_{vo}' = \frac{1}{2} \sin\phi' \, OCR^{\Lambda}$ where $\Lambda = 1-C_s/C_c$
  - Yield surface represents 3-d preconsolidation

Overview: Simplified Critical State Soil Mechanics





Four Basic Stress Paths:

1. *Drained NC (decrease $\Delta V/V_o$)*
2. *Undrained NC (positive Δu)*
3. *Undrained OC (negative Δu)*
4. *Drained OC (increase $\Delta V/V_o$)*