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Critical-State Soil Mechanics For Dummies

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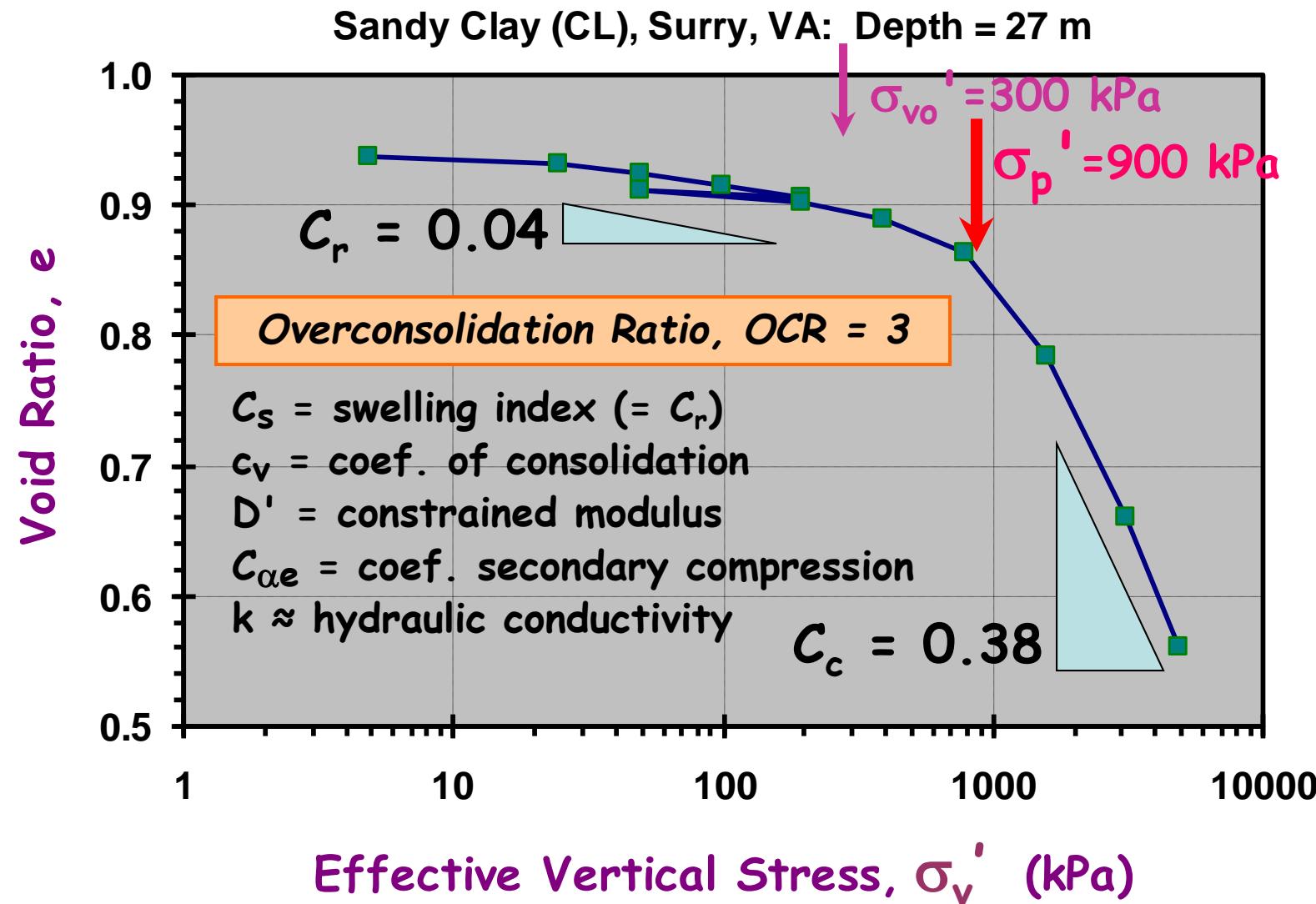
PROLOGUE

- Critical-state soil mechanics is an effective stress framework describing mechanical soil response
- In its simplest form here, we consider only shear-induced loading.
- We merely tie together two well-known concepts: (1) one-dimensional consolidation behavior, represented by e - $\log\sigma'_v$ curves; and (2) shear stress-vs. normal stress ($\tau-\sigma'_v$) from direct shear box or simple shearing (alias Mohr's circles).
- Herein, only the bare essence of CSSM concepts are presented, sufficient to describe strength & compressibility response.

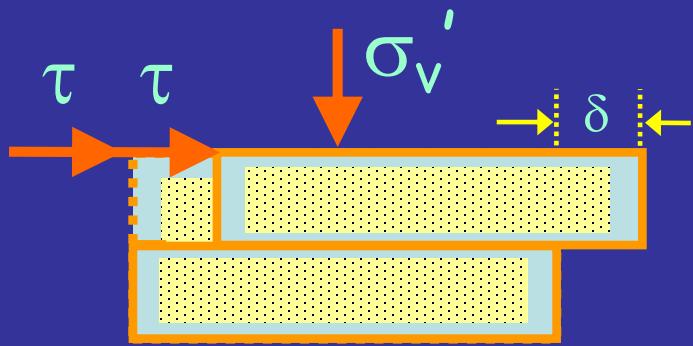
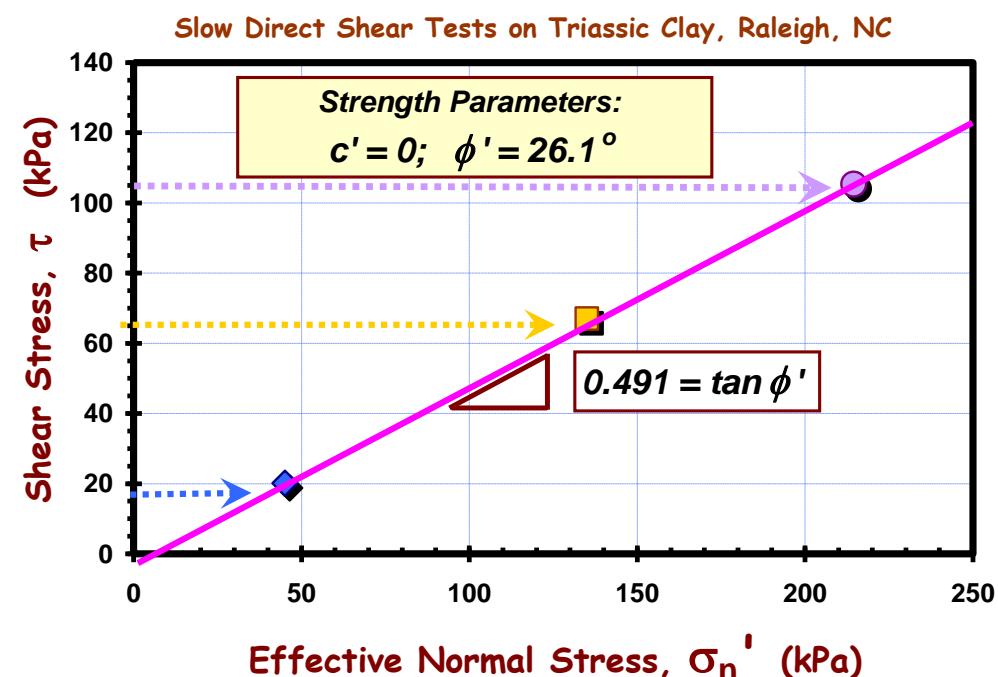
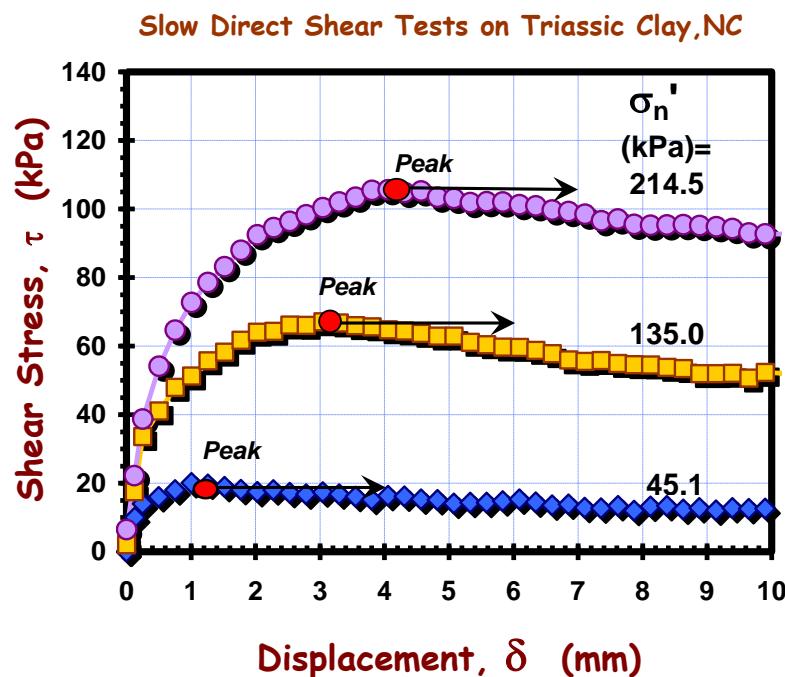
Critical State Soil Mechanics (CSSM)

- Experimental evidence provided by Hvorslev (1936; 1960, ASCE); Henkel (1960, ASCE Boulder) Henkel & Sowa (1961, ASTM STP 361)
- Mathematics presented elsewhere, including: Schofield & Wroth (1968); Burland (1968); Wood (1990).
- In basic form: 3 material constants (ϕ' , C_c , C_s) plus initial state (e_0 , σ_{vo}' , OCR)
- Constitutive Models, include: Original Cam-Clay, Modified Cam Clay, NorSand, Bounding Surface, MIT-E3 (Whittle, 1993) & MIT-S1 (Pestana) and others (Adachi, Oka, Ohta, Dafalias)
- "Undrained" is just one specific stress path
- Yet !!! CSSM is missing from most textbooks and undergrad & grad curricula in the USA.

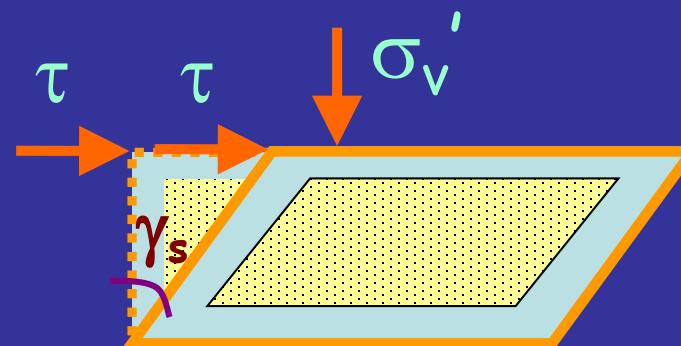
One-Dimensional Consolidation



Direct Shear Test Results

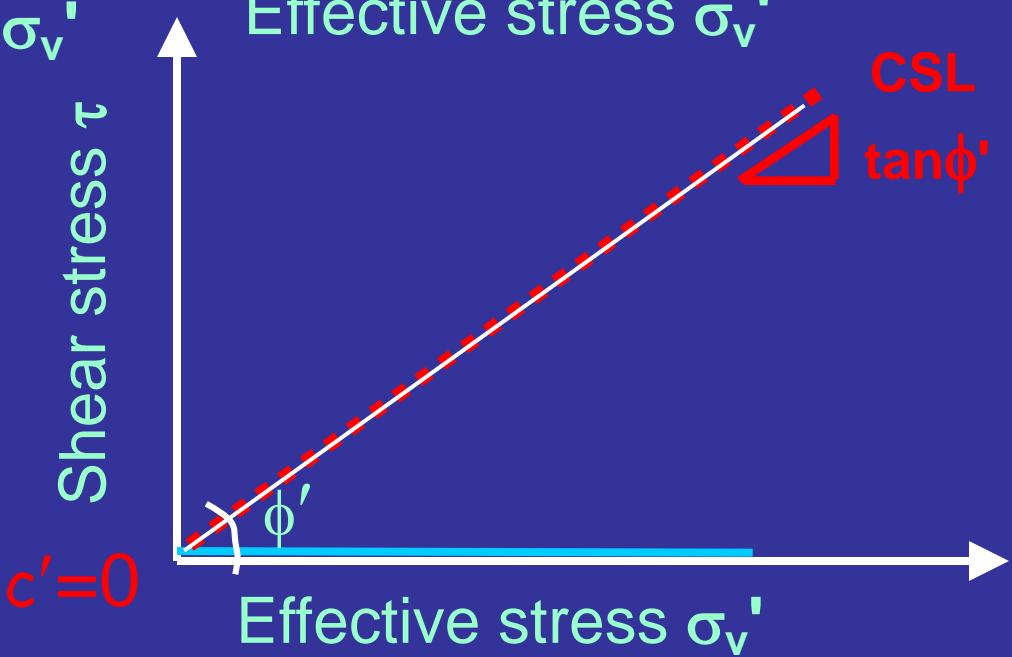
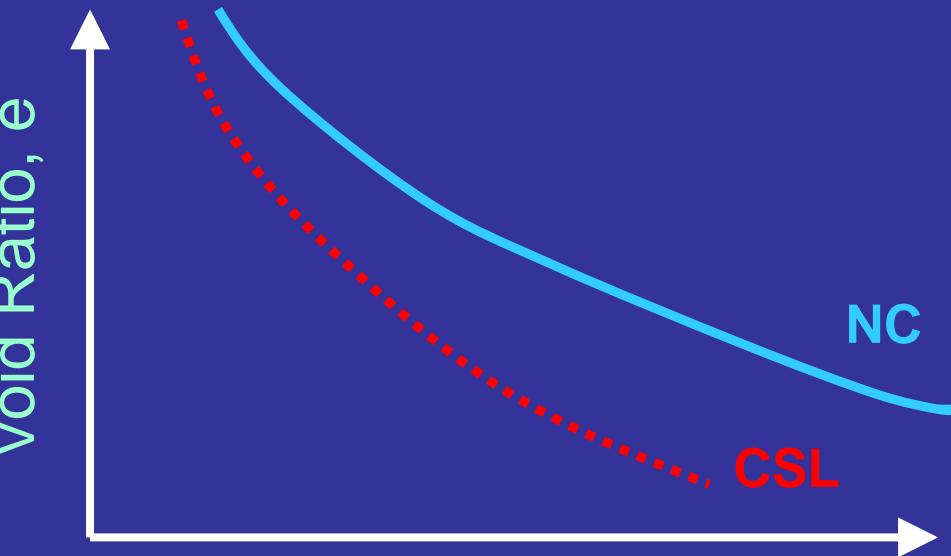
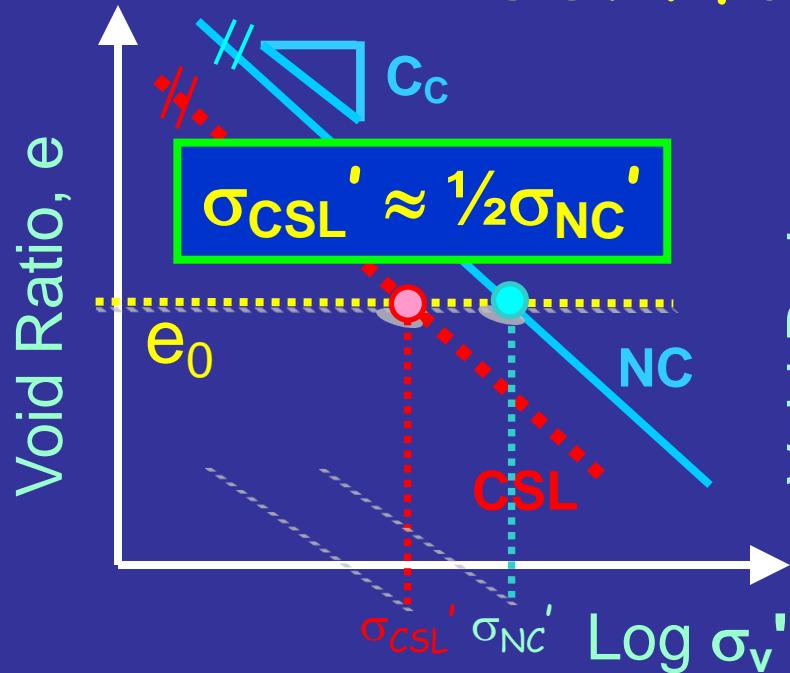


Direct Shear Box (DSB)



Direct Simple Shear (DSS)

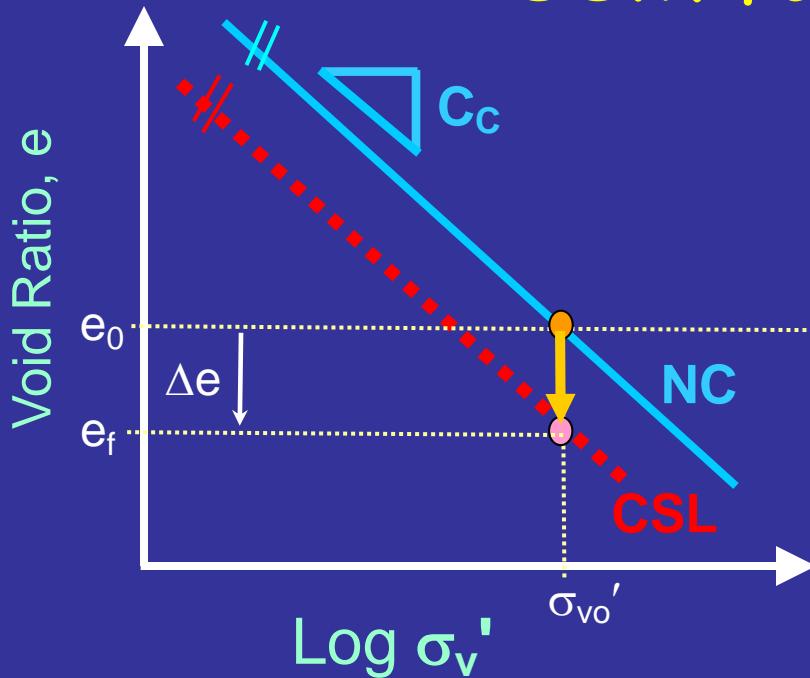
CSSM for Dummies



CSSM Premise:

**"All stress paths fail
on the critical state
line (CSL)"**

CSSM for Dummies



STRESS PATH No.1

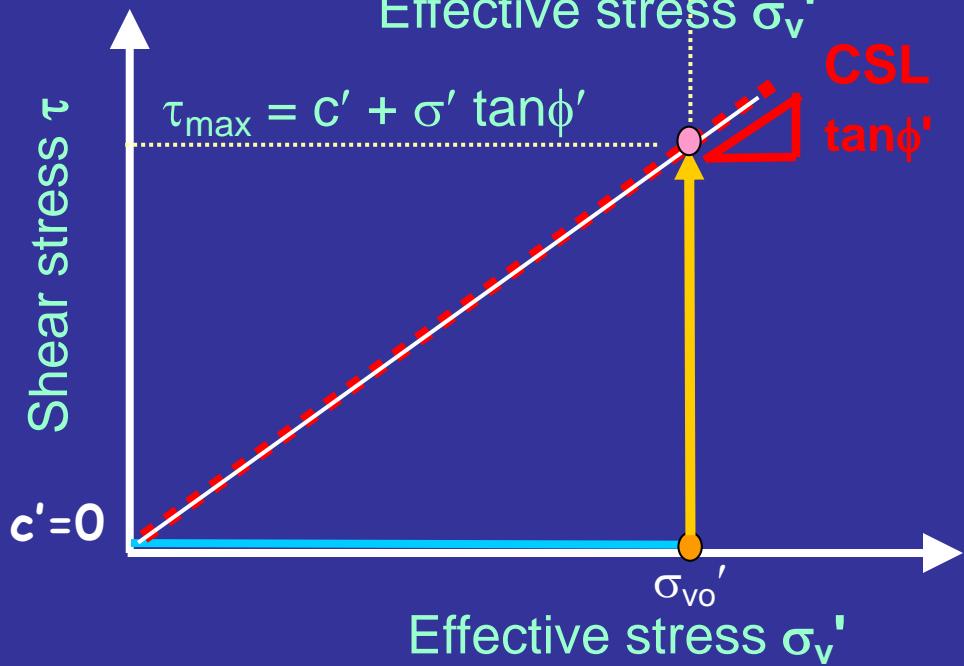
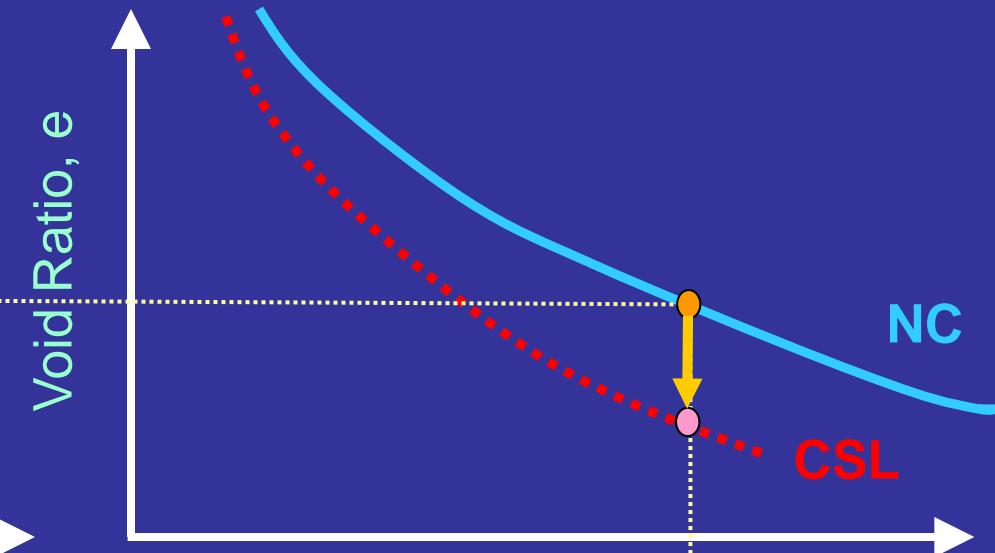
NC Drained Soil

Given: e_0 , σ_{vo}' , NC ($OCR=1$)

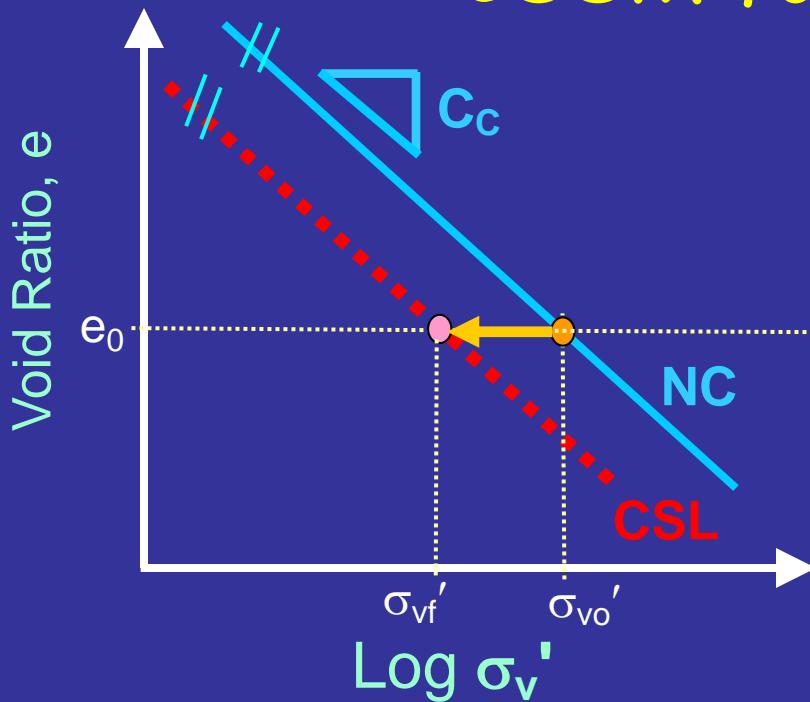
Drained Path: $\Delta u = 0$

Volume Change is Contractive:

$$\varepsilon_{vol} = \Delta e / (1 + e_0) < 0$$



CSSM for Dummies



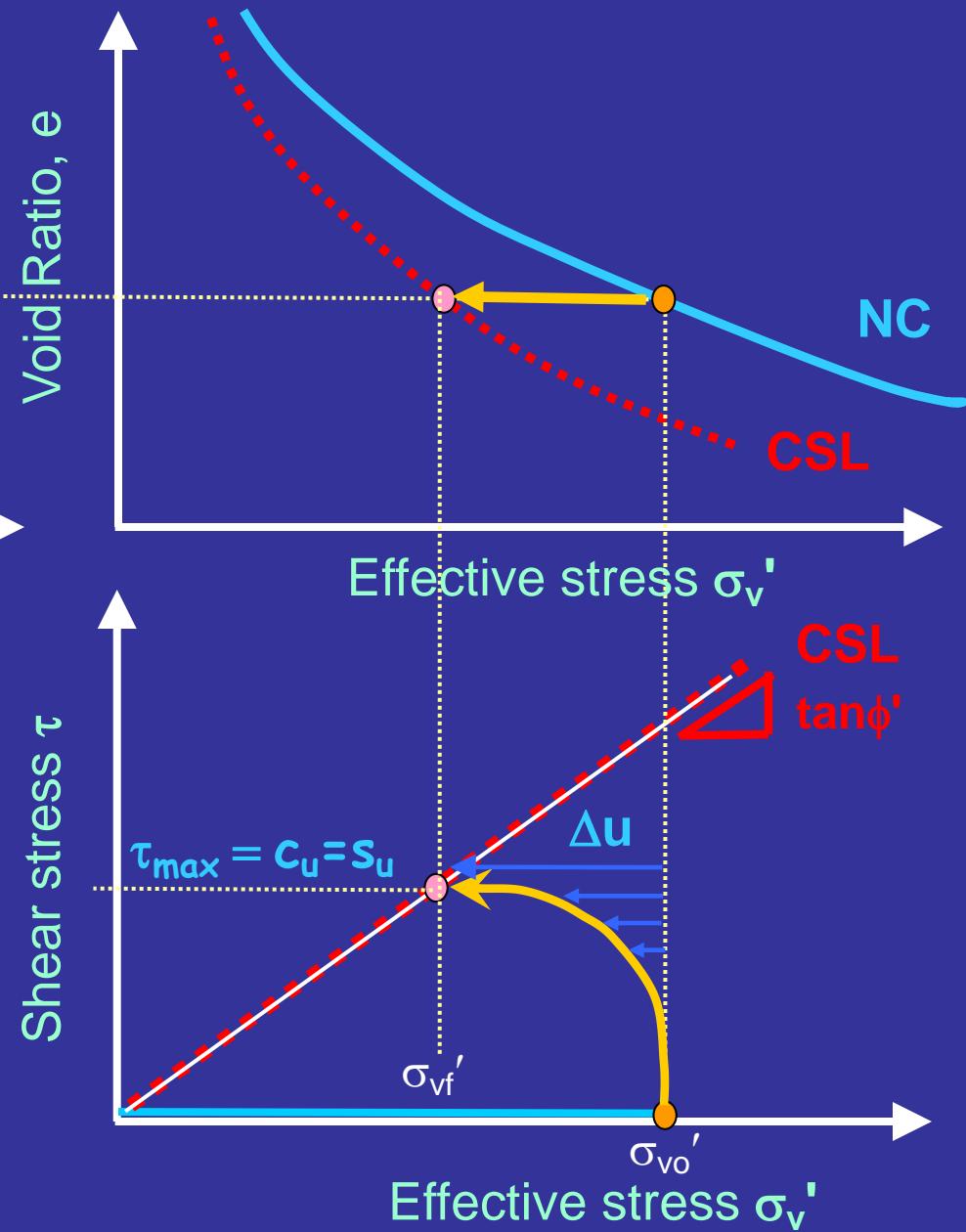
STRESS PATH No.2

NC Undrained Soil

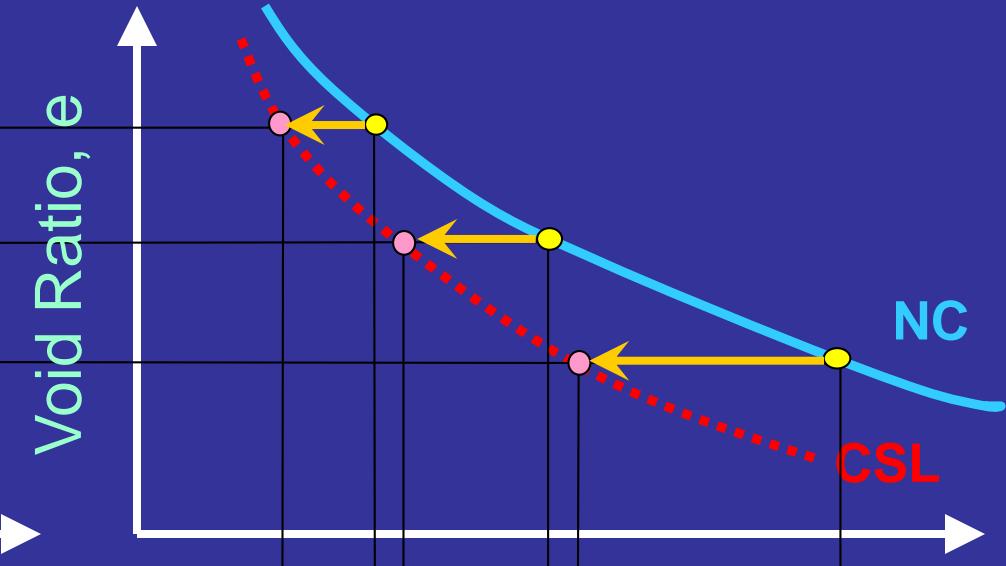
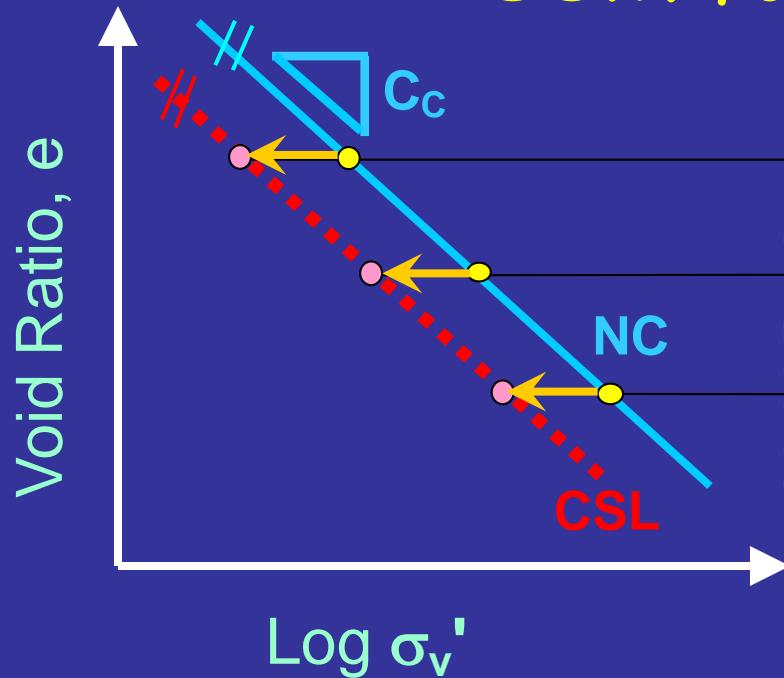
Given: e_0 , σ_{vo}' , NC ($OCR=1$)

Undrained Path: $\Delta V/V_0 = 0$

$+\Delta u$ = Positive Excess
Porewater Pressures

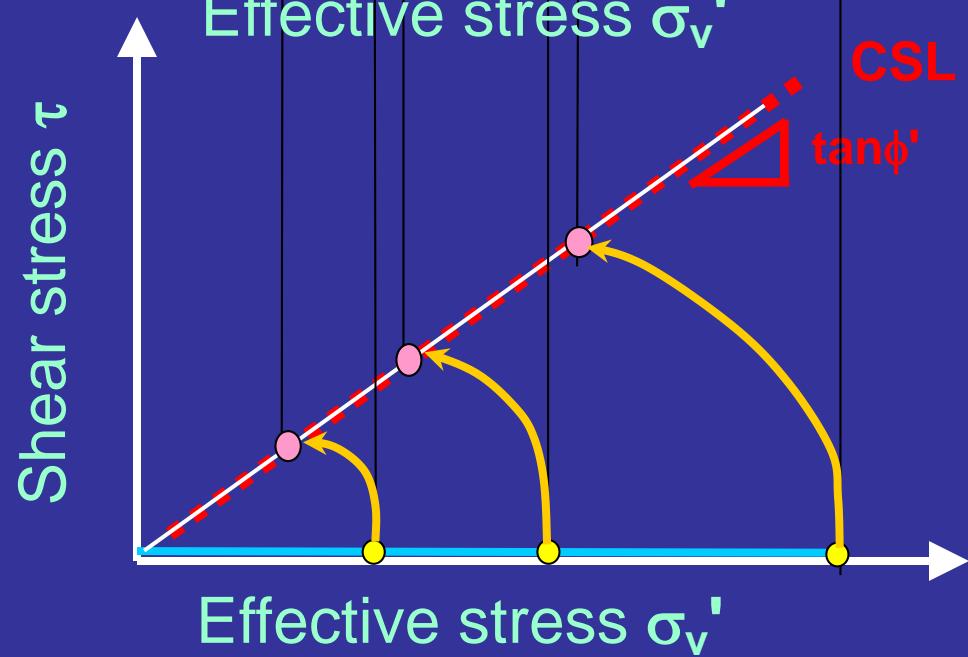


CSSM for Dummies

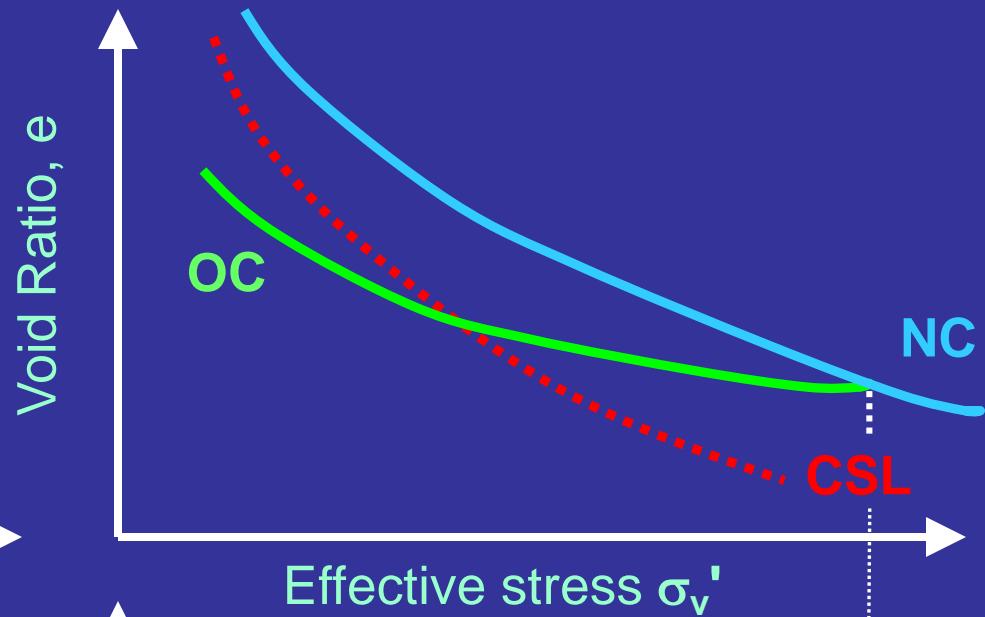
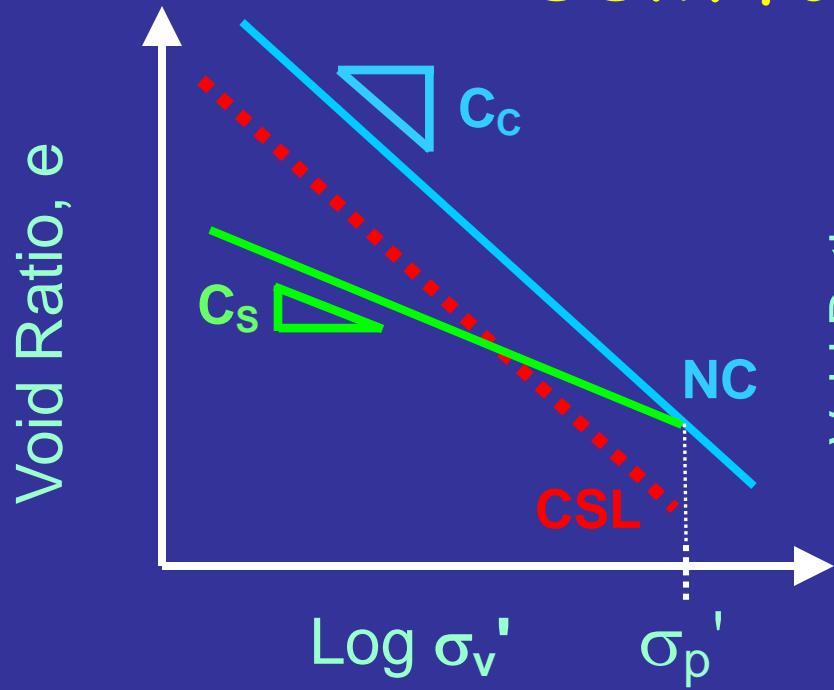


Note: All NC undrained stress paths are parallel to each other, thus:
 $s_u/\sigma_{vo}' = \text{constant}$

$$\text{DSS: } s_u/\sigma_{vo}'_{NC} = \frac{1}{2}\sin\phi'$$



CSSM for Dummies



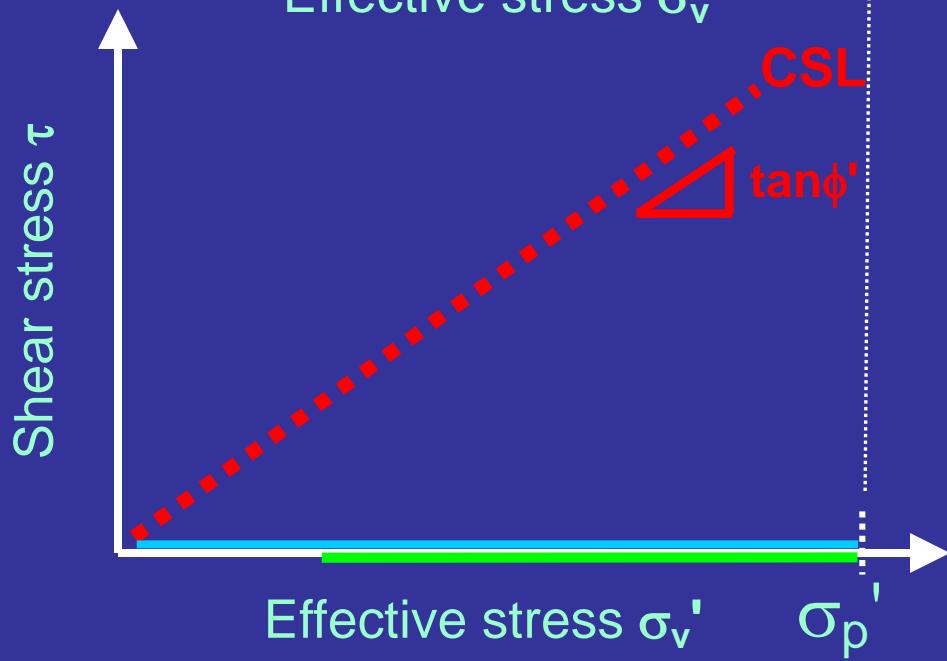
Overconsolidated States:

e_0 , σ_{vo}' , and $OCR = \sigma_p'/\sigma_{vo}'$

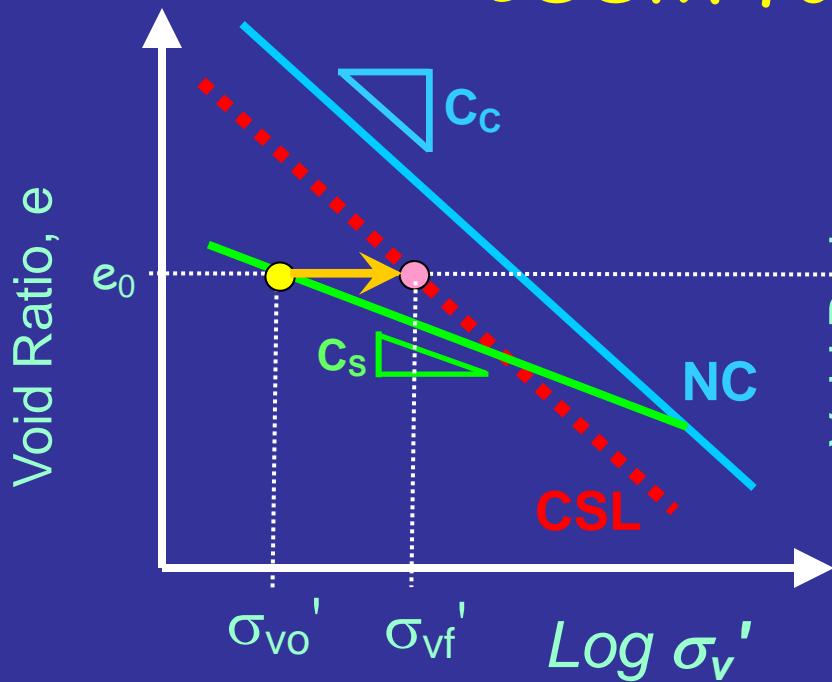
where $\sigma_p' = \sigma_{vmax}' = P_c' =$

preconsolidation stress;

$OCR = \text{overconsolidation ratio}$



CSSM for Dummies



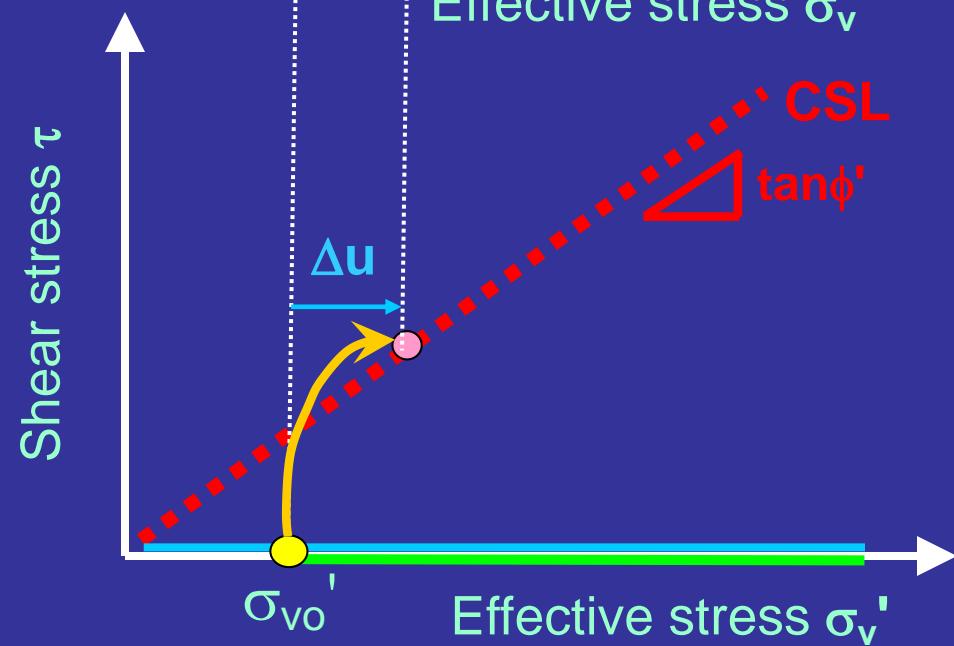
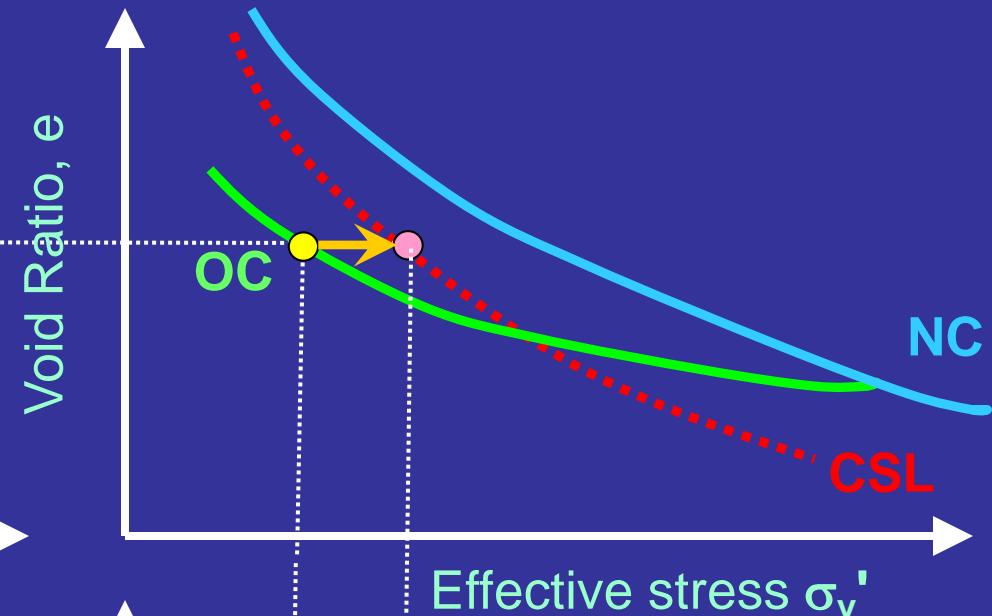
Stress Path No. 3

Undrained OC Soil:

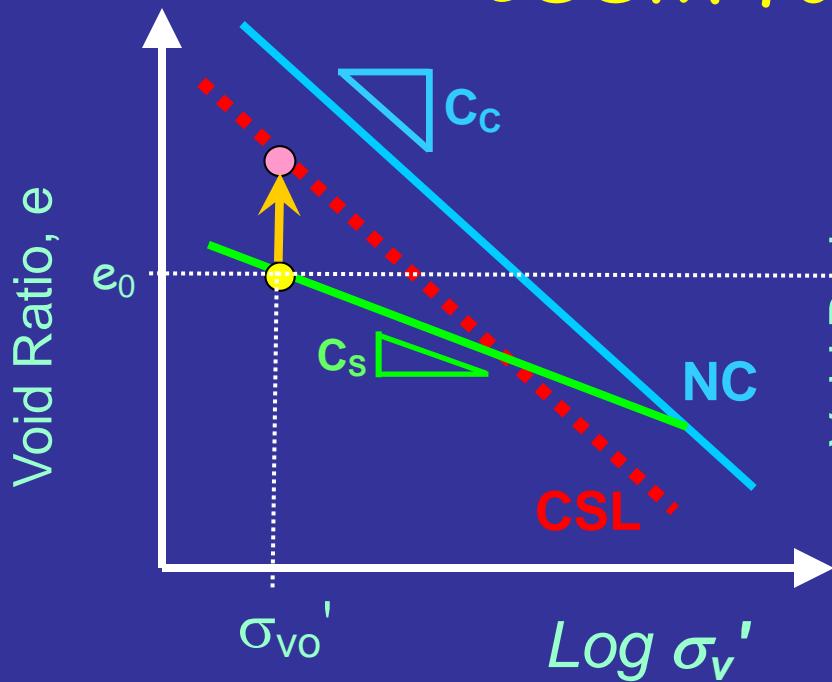
e_0 , σ_{vo}' , and OCR

Stress Path: $\Delta V/V_0 = 0$

Negative Excess Δu



CSSM for Dummies



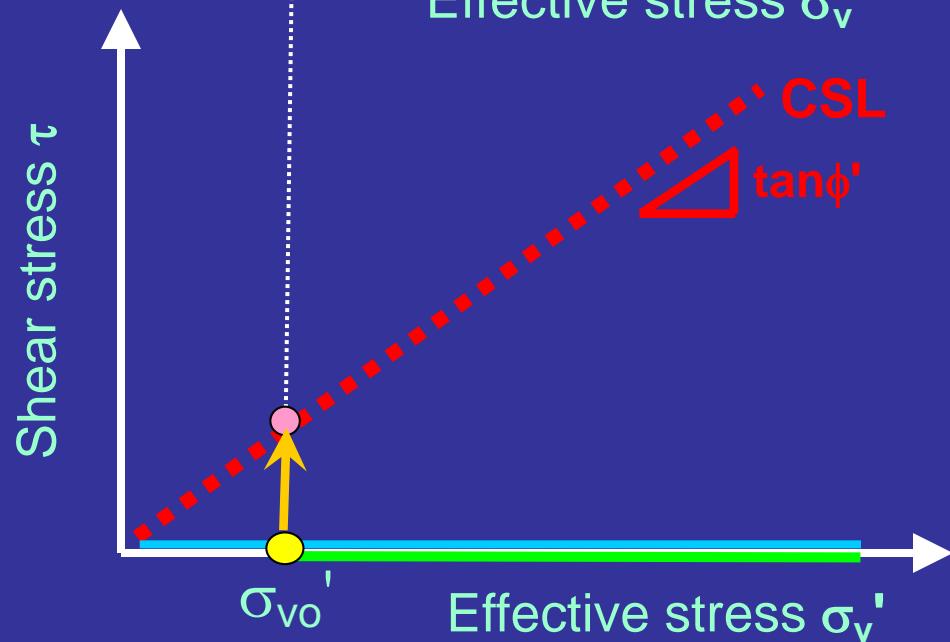
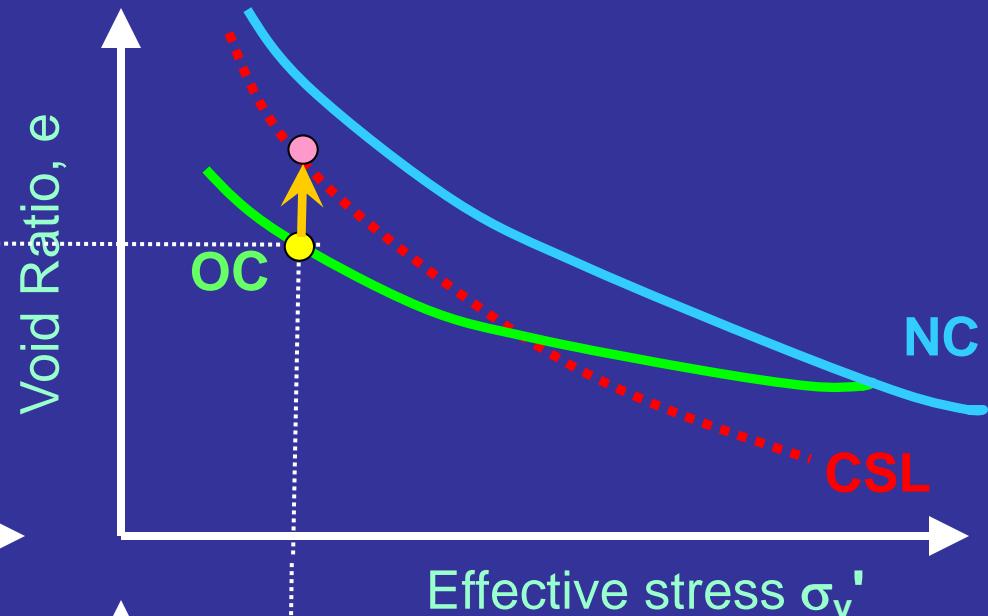
Stress Path No. 4

Drained OC Soil:

e_0 , σ_{vo}' , and OCR

Stress Path: $\Delta u = 0$

Dilatancy: $\Delta V/V_0 > 0$

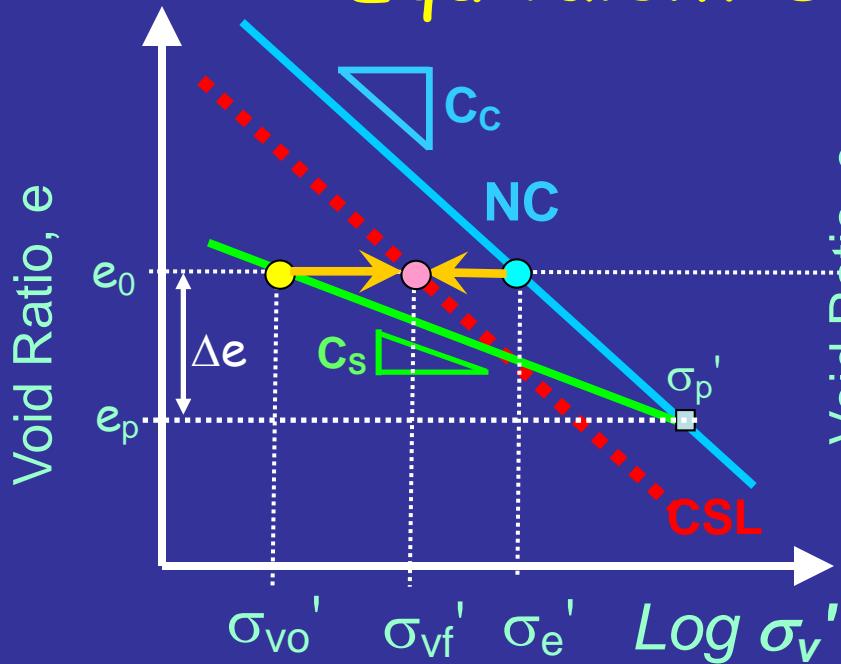


Critical state soil mechanics

- Initial state: e_0 , σ_{vo}' , and $OCR = \sigma_p'/\sigma_{vo}'$
- Soil constants: ϕ' , C_c , and C_s ($\Lambda = 1 - C_s/C_c$)
- For NC soil ($OCR = 1$):
 - Undrained ($\varepsilon_{vol} = 0$): $+ \Delta u$ and $\tau_{max} = s_u = c_u$
 - Drained ($\Delta u = 0$) and contractive (decrease ε_{vol})
- For OC soil:
 - Undrained ($\varepsilon_{vol} = 0$): $- \Delta u$ and $\tau_{max} = s_u = c_u$
 - Drained ($\Delta u = 0$) and dilative (Increase ε_{vol})

There's more ! Semi-drained, Partly undrained, Cyclic.....

Equivalent Stress Concept

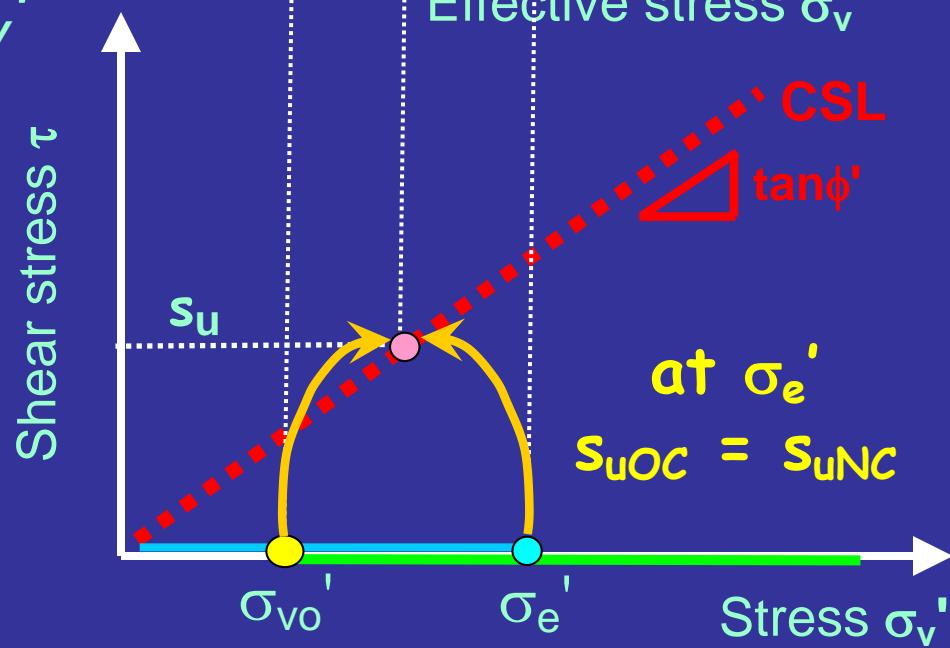
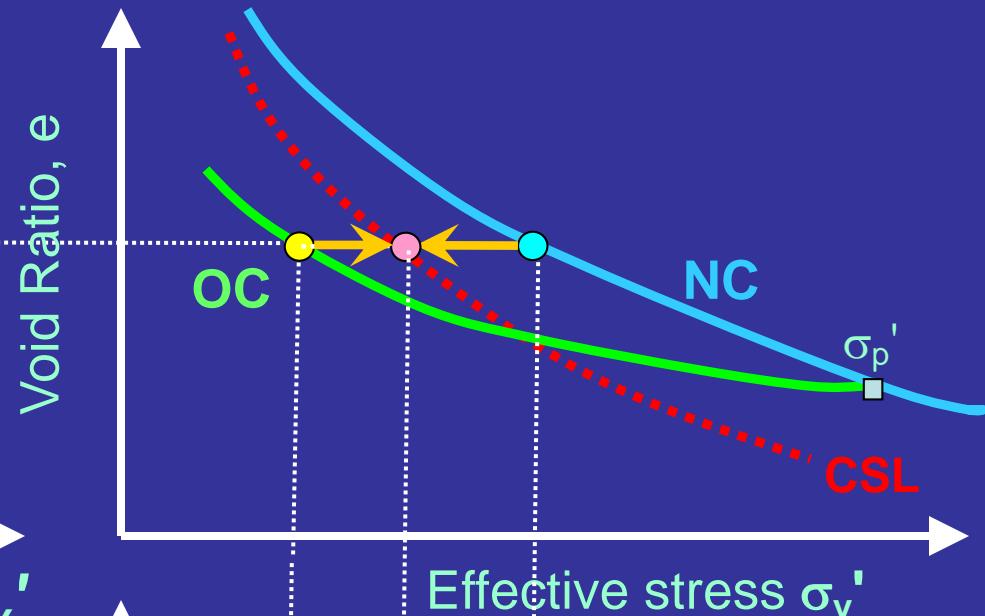


1. OC State (e_0 , σ_{vo}' , σ_p')
2. Project OC state to NC line for equivalent stress, σ_e'

$$\Delta e = C_s \log(\sigma_p' / \sigma_{vo}')$$

$$\Delta e = C_c \log(\sigma_e' / \sigma_p')$$

$$3. \quad \sigma_e' = \sigma_{vo}' OCR^{[1 - Cs/Cc]}$$



Critical state soil mechanics

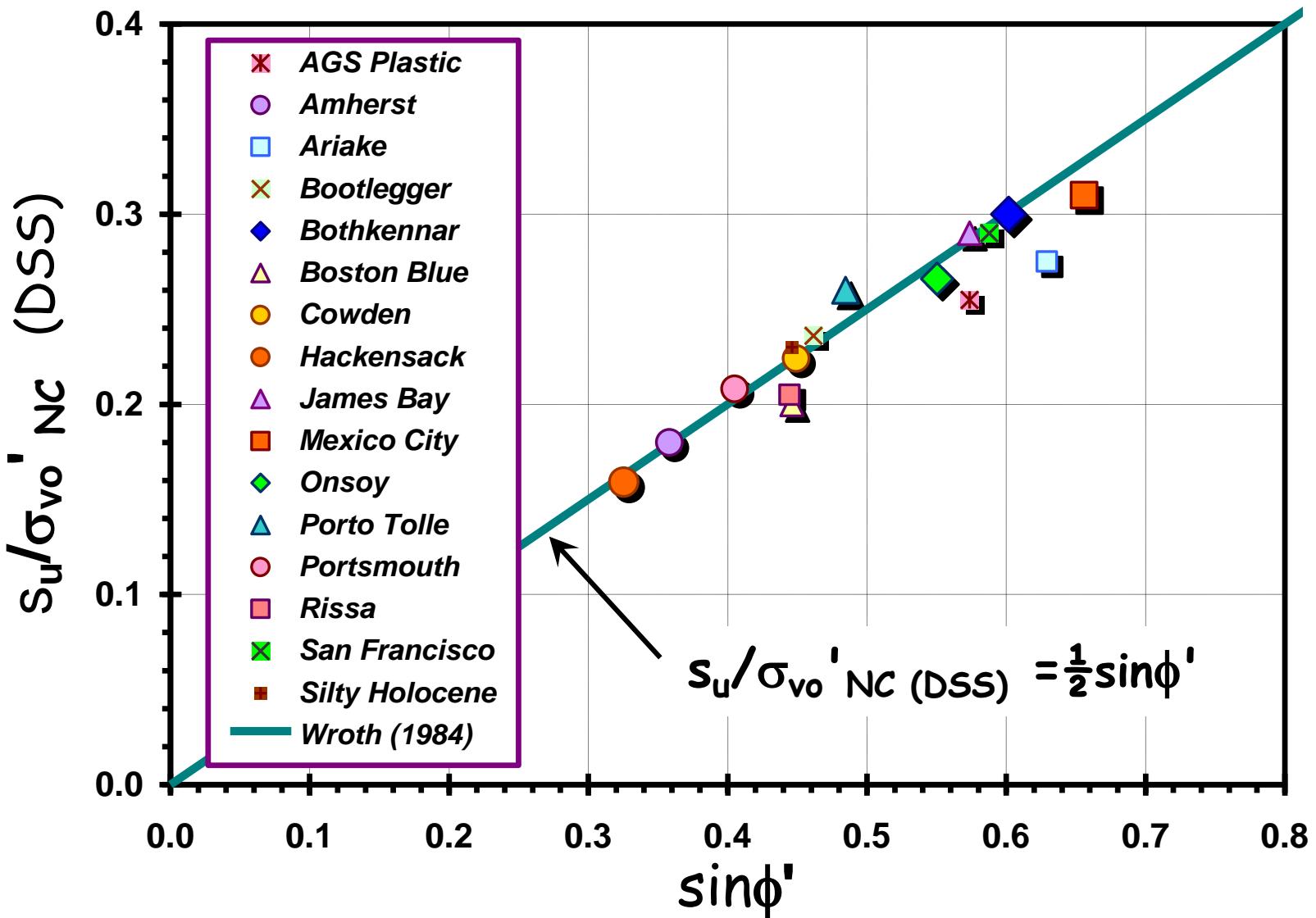
- Previously: $s_u/\sigma_{vo}' = \text{constant}$ for NC soil
- On the virgin compression line: $\sigma_{vo}' = \sigma_e'$
- Thus: $s_u/\sigma_e' = \text{constant}$ for all soil (NC & OC)
- For simple shear: $s_u/\sigma_e' = \frac{1}{2}\sin\phi'$
- Equivalent stress: $\sigma_e' = \sigma_{vo}' \text{OCR}^{[1-C_s/C_c]}$

Normalized Undrained Shear Strength:

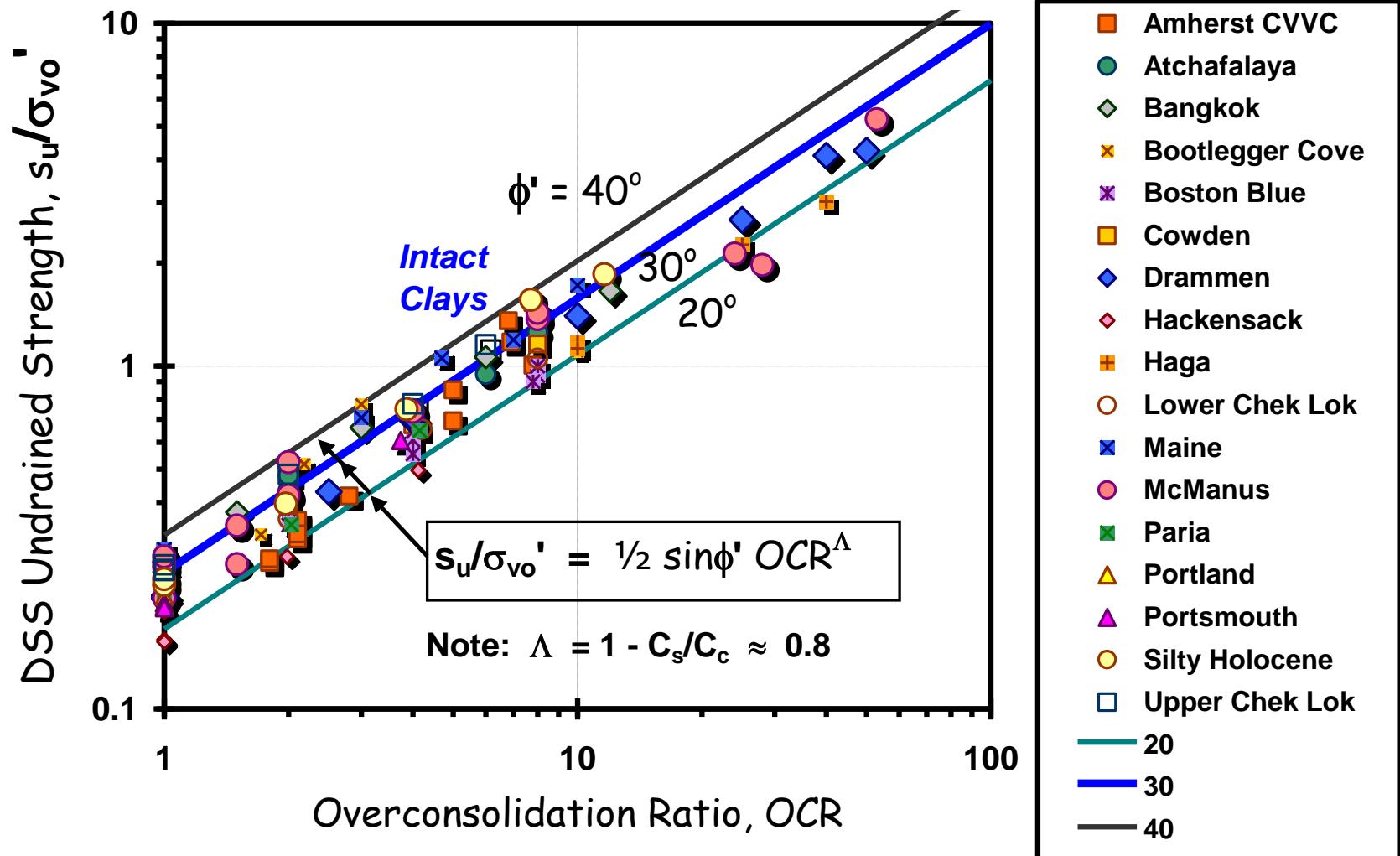
$$s_u/\sigma_{vo}' = \frac{1}{2} \sin\phi' \text{OCR}^{\Lambda}$$

$$\text{where } \Lambda = (1 - C_s/C_c)$$

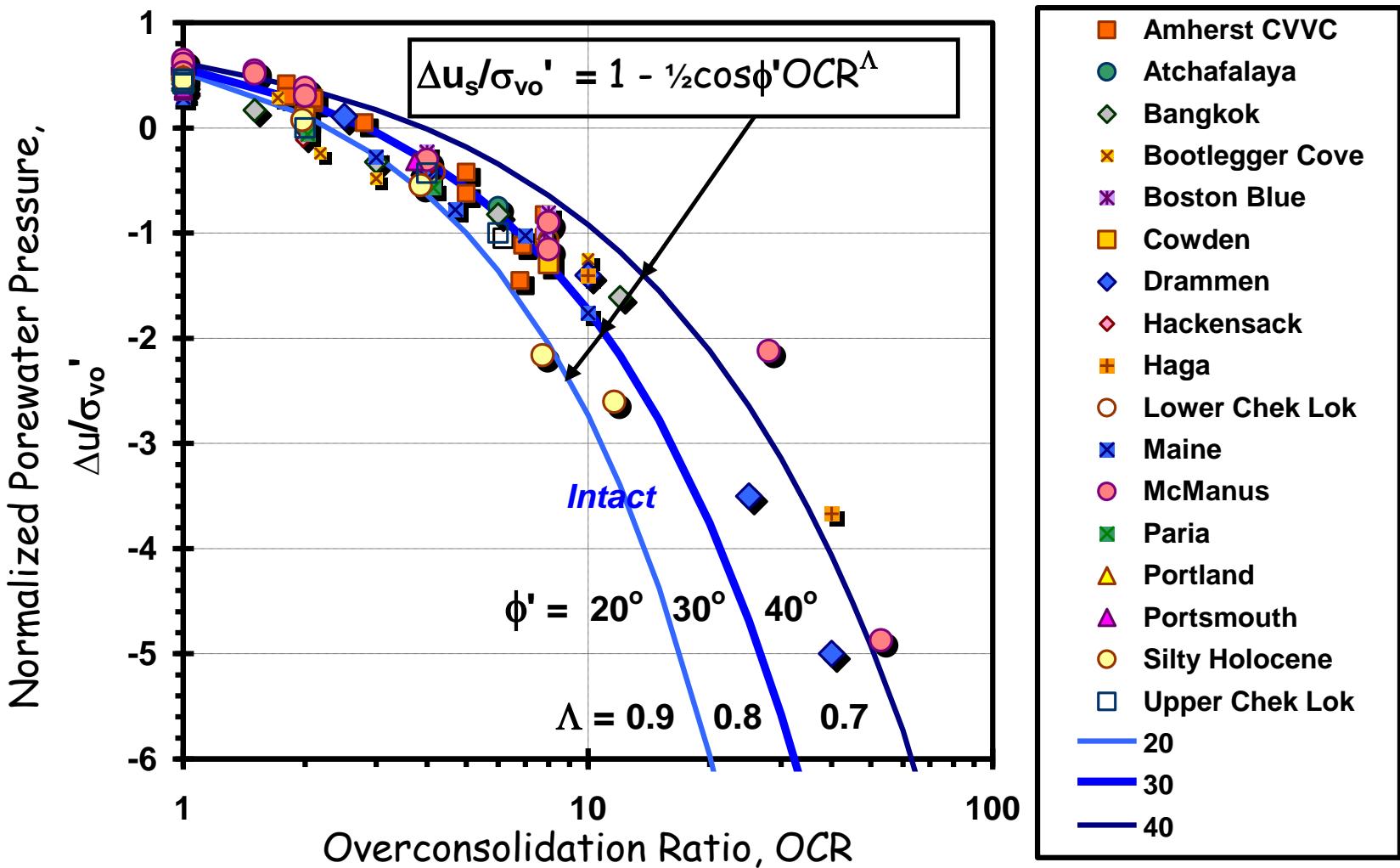
Undrained Shear Strength from CSSM



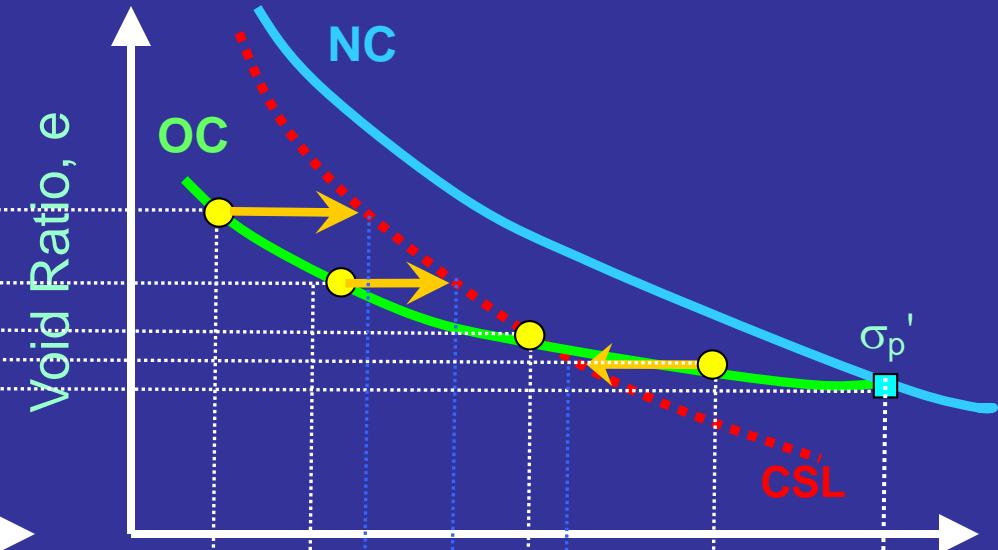
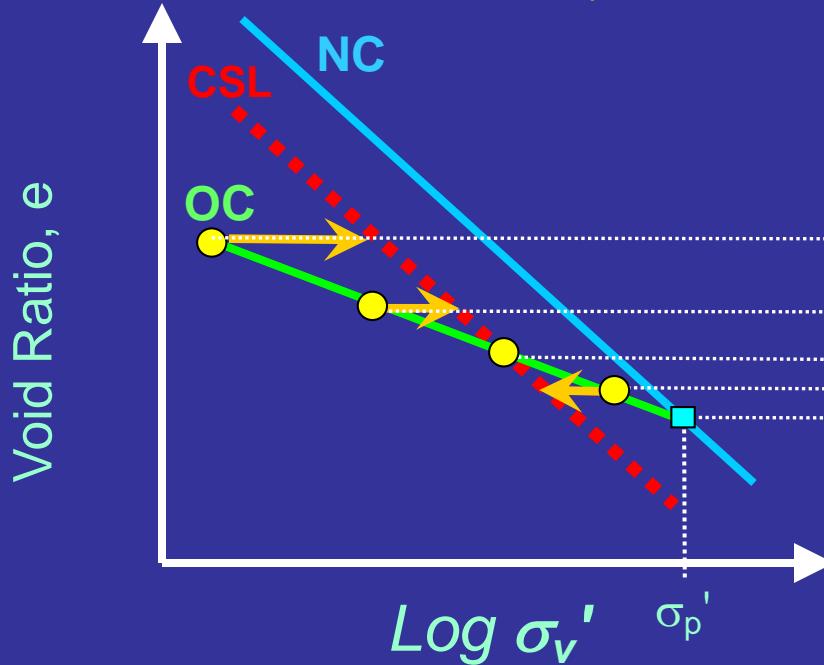
Undrained Shear Strength from CSSM



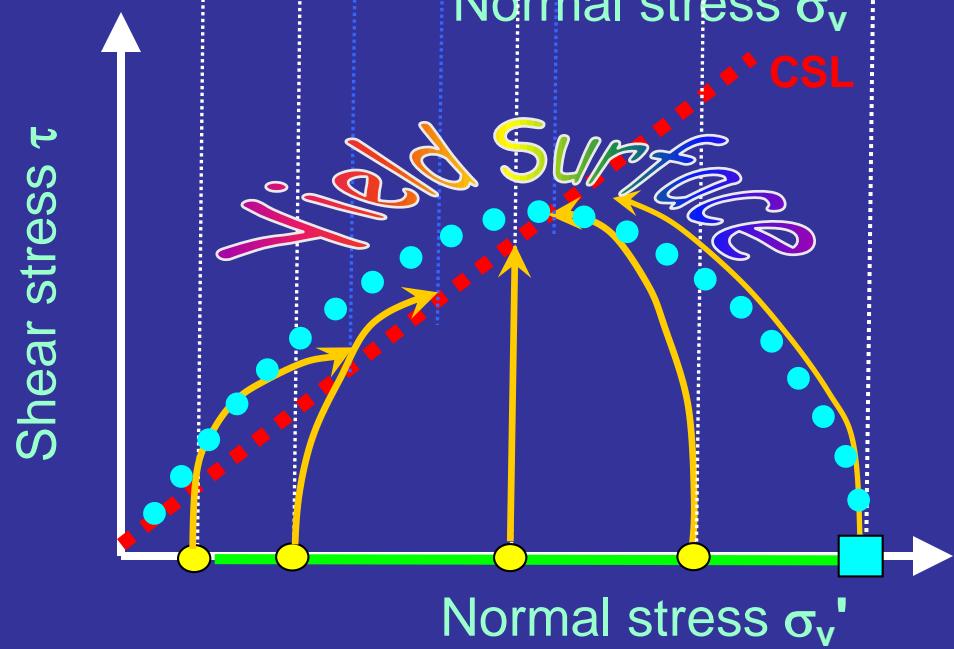
Porewater Pressure Response from CSSM



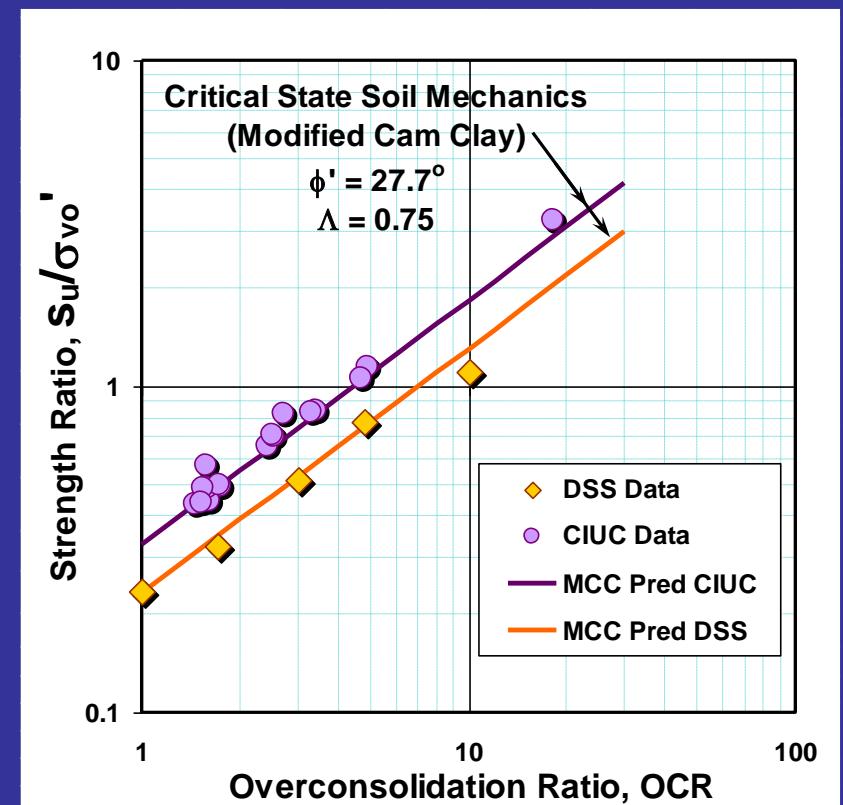
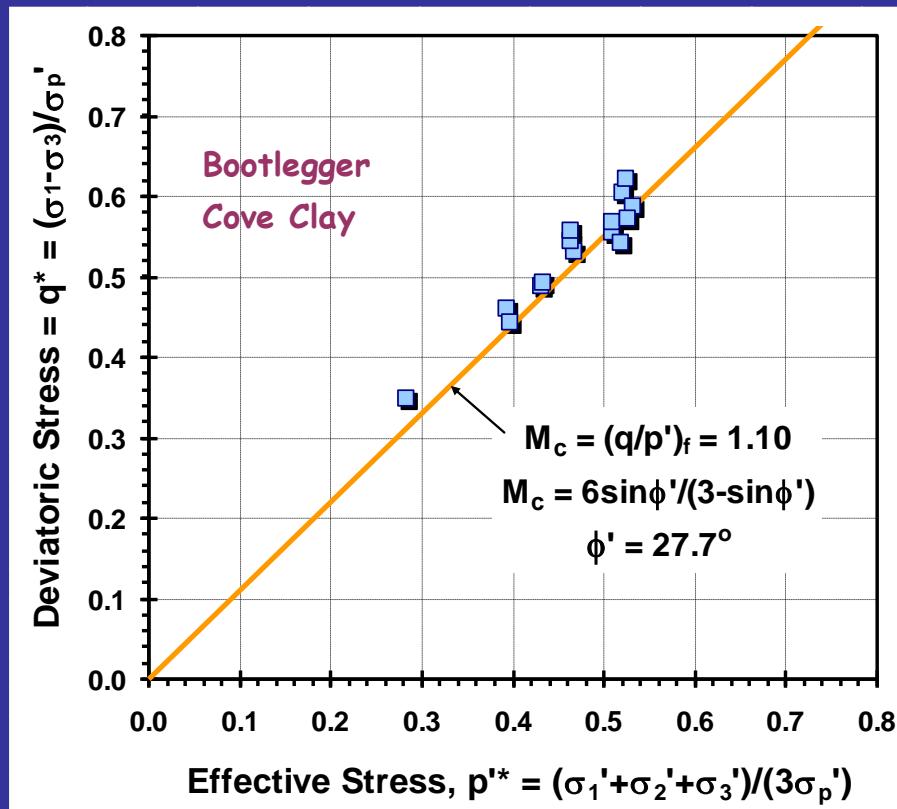
Yield Surfaces



- Yield surface represents 3-d preconsolidation
- Quasi-elastic behavior within the yield surface



Port of Anchorage, Alaska

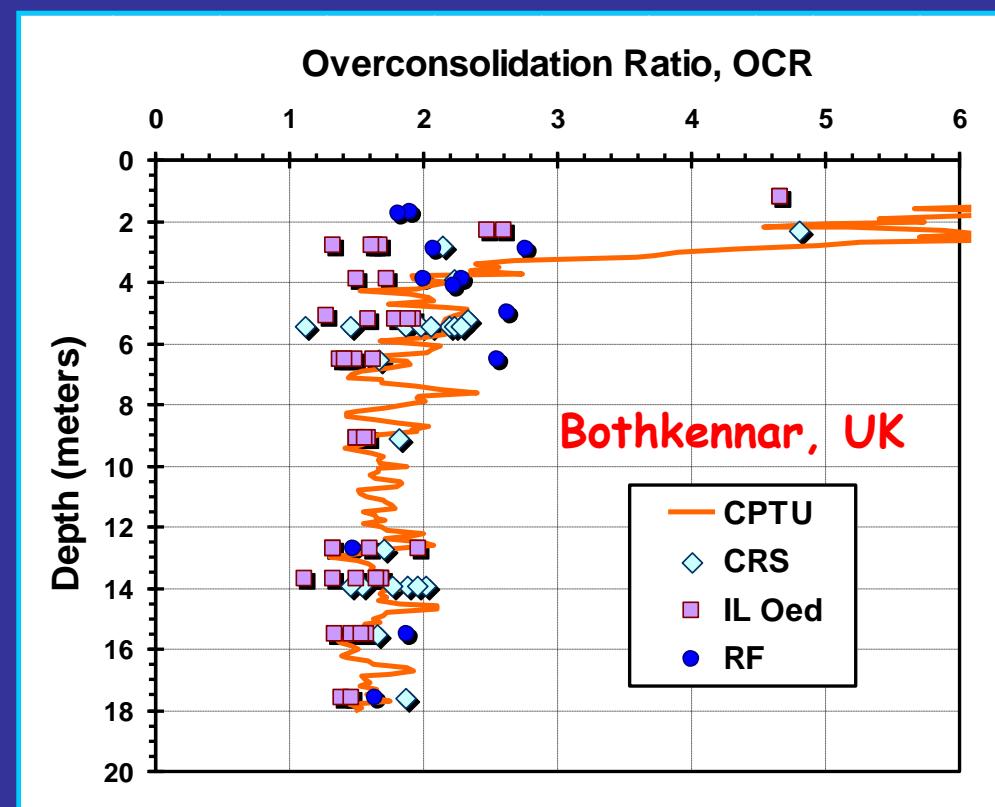
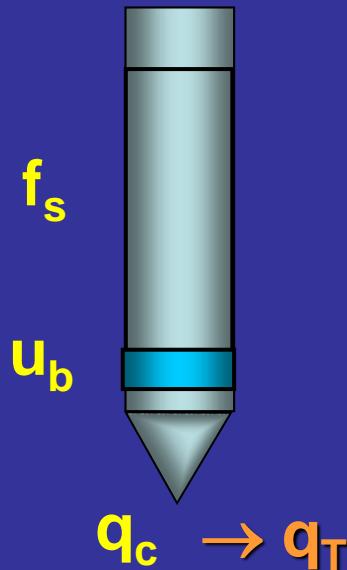


Cavity Expansion - Critical State Model for Evaluating OCR in Clays from Piezocone Tests

$$OCR = 2 \left[\frac{1}{1.95M + 1} \left(\frac{q_T - u_b}{\sigma_{vo}} \right) \right]^{1/\Lambda}$$

where $M = 6 \sin\phi' / (3 - \sin\phi')$

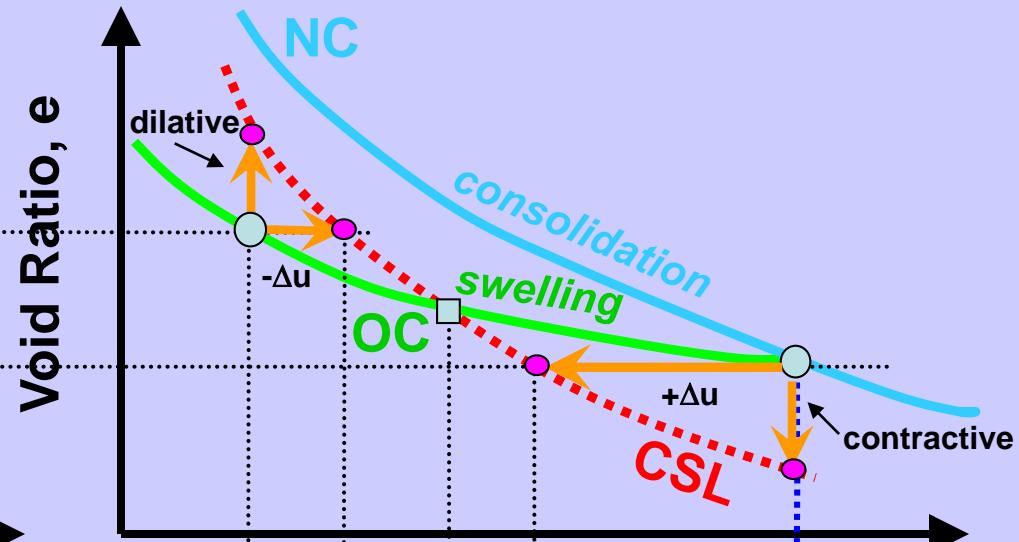
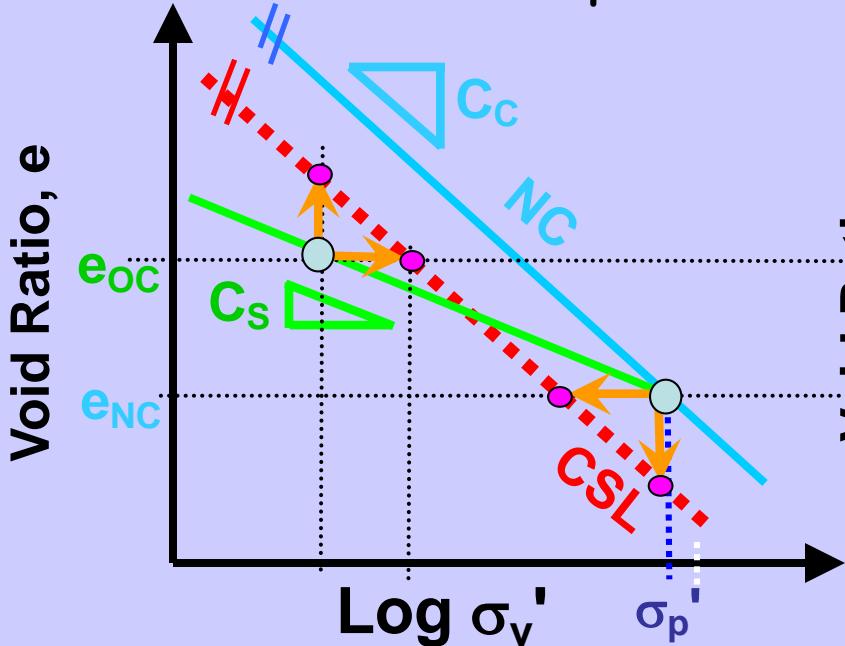
and $\Lambda = 1 - C_s/C_c \approx 0.8$



Critical state soil mechanics

- Initial state: e_0 , σ_{vo}' , and $OCR = \sigma_p'/\sigma_{vo}'$
- Soil constants: ϕ' , C_c , and C_s ($\Lambda = 1 - C_s/C_c$)
- Using effective stresses, CSSM addresses:
 - NC and OC behavior
 - Undrained vs. Drained (and other paths)
 - Positive vs. negative porewater pressures
 - Volume changes (contractive vs. dilative)
 - $s_u/\sigma_{vo}' = \frac{1}{2} \sin\phi' OCR^\Lambda$ where $\Lambda = 1 - C_s/C_c$
 - Yield surface represents 3-d preconsolidation

Overview: Simplified Critical State Soil Mechanics



Four Basic Stress Paths:

1. Drained NC (decrease $\Delta V/V_o$)
2. Undrained NC (positive Δu)
3. Undrained OC (negative Δu)
4. Drained OC (increase $\Delta V/V_o$)

